Introduction to Modal Logic

Introduction

UNI FREIBURG

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Modalities and modal logic

What is a modality?

1: . . .
2: the classification of logical propositions according to their asserting or denying the possibility, impossibility, contingency, or necessity of their content
3/4: . . .

(from *Merriam-Webster's Online Dictionary*)

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Modalities and modal logic

Modal logic?

Modal logic can be viewed broadly as the logic of different sorts of modalities, or modes of truth: alethic (“necessarily”), epistemic (“it is known that”), deontic (“it ought to be the case that”), or temporal (“it has been the case that”) among others . . . In the strict sense however, the term “modal logic” is reserved for the logic of the alethic modalities.

(from *Stanford Encyclopedia of Philosophy*)

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Basic modal logic

Modal operators:

$\Box\varphi$ it is necessarily the case that φ

$\Diamond\varphi$ it is possibly the case that φ

Examples:

- $\Diamond\Box\Diamond\varphi$
- $\Diamond(\varphi \wedge \Diamond\psi)$
- $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$

Question:

Which formulae are satisfiable, which are logically valid?

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Basic modal logic

Modal operators:

$\Box\varphi$ it is necessarily the case that φ

$\Diamond\varphi$ it is possibly the case that φ

Interpretation:

- relational structures
- topological spaces
- modal algebras

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Wise men puzzle

The story ...

A king wishes to test his three wise men. The three wise men are arranged in a circle so that they can see and hear each other. The king announces to them that there are three red hats and two white hats. The king puts a hat on the head of each of the three wise men and asks them sequentially: “Do you know the color of the hat on your head?”.
The first and then the second wise men answer that they do not know, but the third answers that he does know.
Which color has the hat of the third wise man?

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Example: Doxastic / epistemic logic

Modal operators:

$B_a\varphi$	agent a holds φ true
$B_a(\varphi, \psi)$	ψ is a reason for a to believe φ
$K_a\varphi$	agent a knows φ
$K\varphi$	it is known that φ is true
$C_\gamma\varphi$	among the members of group γ it is common knowledge that φ

Interpretation:

- relational structures
- belief sets (set filters)
- probability functions

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Example: Deontic logic

Modal operators:

$O_a\varphi$	agent a is obliged to make φ true
$O_a(\varphi, \psi)$	given ψ, a is obliged to make φ true
$O\varphi$	φ should be true
$A\varphi$	φ is allowed
$F\varphi$	φ is forbidden

Interpretation:

- relational structures (normatively optimal worlds?)
- utility functions

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Example: Temporal logic

Modal operators:

$\mathsf{F}\varphi$	sometimes in the future φ will be true
$\mathsf{G}\varphi$	always in the future φ will be true
$\mathsf{X}\varphi$	in the next state φ will be true
$\mathsf{U}(\varphi,\psi)$	until φ will be true, ψ will be true

Interpretation:

- discrete / continuous time
- linear / branching time
- concrete flows of time $\langle \mathbb{N}, <_{\mathbb{N}} \rangle$, $\langle \mathbb{Q}, <_{\mathbb{Q}} \rangle$, $\langle \mathbb{R}, <_{\mathbb{R}} \rangle$

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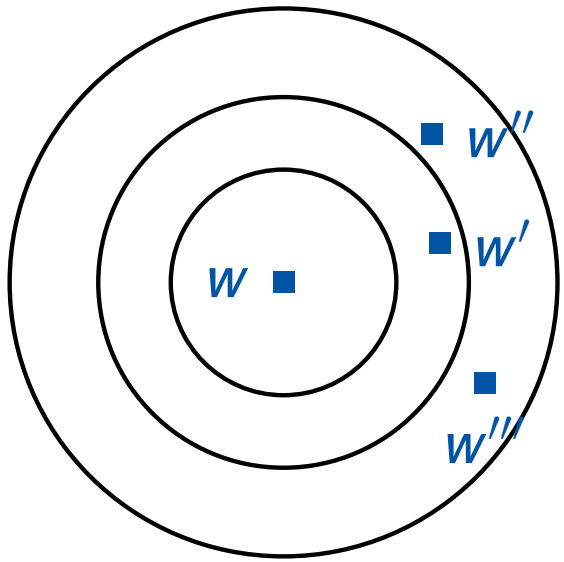
Example: Conditional logics

Modal operators:

$\varphi \Box\!\!\rightarrow \psi$ if φ, then ψ

Interpretation:

- sphere models based on similarity relation between words (Lewis, 1973)



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Example: Probabilistic modal logics

Modal operators:

$\Box_p \varphi$ in the next step, φ will be true with probability $\geq p$

Interpretation (Larsen and Skou, 1991):

- probabilistic transition systems

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Example: Agent logics

Modal operators:

$[\gamma]\varphi$ the group of agents, γ, has a joint strategy to guarantee that φ becomes true

Interpretation:

- non-deterministic transition systems

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Example: Dynamic logics

Modal operators:

$[\alpha]\varphi$ after executing α, φ will be true

$[\alpha;\beta]\varphi$ after executing α followed by β, φ will be true

...

$[(\psi?;\alpha)\cup(\neg\psi?;\beta)]\varphi$

after executing the program "If ψ, then α; else β", φ will be true

...

Interpretation:

- labeled transition systems

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Example: Description logic

"Modal" operators:

$\exists r.\varphi$	true for all objects x such that there exists an object y that is in relation r to x and satisfies φ
$\forall r.\varphi$	true for all objects x such that for each object y in relation r to x, φ is true for y
...	
$\exists r^{-1}.\varphi$	(converse relation)
$\exists r^{*}.\varphi$	(reflexive and transitive closure)
$\exists (r \circ s).\varphi$	(composition)

Interpretation:

- relational structures / first-order structures

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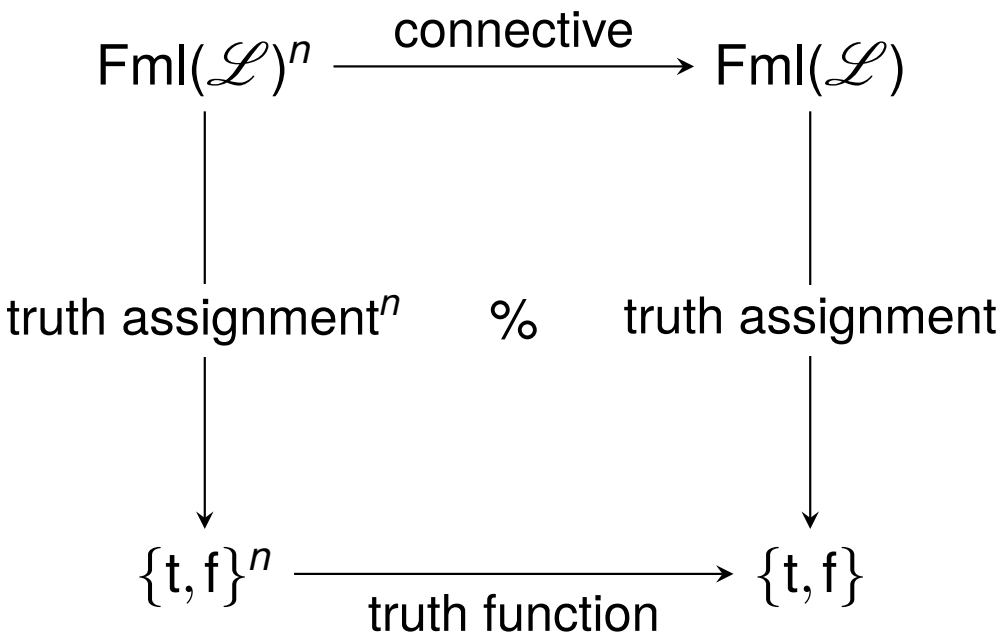
Intensional connectives

A first slogan

Modal logic is the theory of non truth-functional connectives.



Truth-functional connectives

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Relational structures

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Slogan

Modal languages are simple yet expressive languages for talking about relational structures. [BRV02]

- Modal languages extend propositional logic by further connectives (boxes $\Box$ and diamonds $\Diamond$)
- . . . which are semantically characterized in terms of relational structures (e.g., linear orders, transition systems)

Locality

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Slogan

Modal languages provide an internal, local perspective on relational structures. [BRV02]

- Modal formulae are evaluated at a state / possible world in a relational structure / transition system
- . . . and for evaluating a formulae only those states are relevant that are accessible by a transition in the system

Modal logics and other logics

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Slogan

Modal languages are not isolated formal systems. [BRV02]

- Modal languages have corresponding (first- / second-order) languages that describe the same class of relational structures / transition systems
- . . . which enables import and export of results (correspondence theory)

Modal logics in computer science

- Specification languages for properties of reactive and distributed programs:
  temporal logics (LTL, CTL, CTL*), dynamic logics, ...
- Logics for different topics in AI:
  temporal logics, logics of belief/knowledge, logics of space
- Language for data and knowledge representation:
  description logics

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Model checking (verification)

- possible worlds = states of the system that we are verifying
- accessibility/reachability relation = transitions the system can make
- Modal logics are specification languages for expressing the correctness properties the system has to satisfy
  - p will hold in every future state of the system: Gp
  - p will hold infinitely often: GFp
  - Whenever p is true, q will be true later: $G(p \rightarrow Fq)$

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Artificial Intelligence

- Applications similar to those in philosophical logic, except that the goal is to formalize and solve problems by automated reasoning (complexity issues)
- How to reason about knowledge and beliefs?
- How to reason about time and change?
- How to reason about space?

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History of modal logic

- Modal-logical principles presumably first discussed in a systematic way by Aristotle in *De Interpretatione*
  For example:
  - "Necessity implies possibility"
  - Inter-definability of "it is possible that" and "it is necessary that"
  - Logical laws of modalities
- Contributions by the Megarians, the Stoics, Ockham, and Pseudo-Scotus
- Leibniz introduced the concept of possible world

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History of modal logic

Anselm's argument for the existence of God (*Proslogion*, 1077/78, citation: Internet Encyclopedia of Philosophy):

[Even a] fool, when he hears of … a being than which nothing greater can be conceived … understands what he hears, and what he understands is in his understanding. … And assuredly that, than which nothing greater can be conceived, cannot exist in the understanding alone. For suppose it exists in the understanding alone: then it can be conceived to exist in reality; which is greater. … Therefore, if that, than which nothing greater can be conceived, exists in the understanding alone, the very being, than which nothing greater can be conceived, is one, than which a greater can be conceived. But obviously this is impossible. Hence, there is no doubt that there exists a being, than which nothing greater can be conceived, and it exists both in the understanding and in reality.

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History of modal logic

- In the 1910s C. I. Lewis investigated modal logic as a possibility to introduce a strengthening of material implication.
- First semantics of modal logic introduced by A. Tarski (topological semantics)
- R. Carnap's suggestion (1942, 1947): Let M be a set of state descriptions. Then 'Necessarily p' is true in s if and only if p is true in every state description in M.
- A. Prior (1957): 'It was once the case that p' is true at instant t if and only if there is an instant t' earlier than t s. t. p is true at t'.
- S. Kripke (1959) introduced the concept of accessibility relation defined on (possible) worlds. 'Necessarily p' is true in w if and only if p is true in every world w' accessible from w.

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Topics

1. Overview on diverse modal logics
2. The modal logic S5: From propositional to modal logic
3. Modal logic, language and semantics: the general setting
4. Bisimulation and expressivity
5. Completeness and compactness
6. Complexity of reasoning in modal logics
7. Decision procedures for modal logics
8. Doxastic and epistemic modal logics
9. Temporal logics
10. Propositional dynamic logic
11. Fixpoint logics, model μ-calculus
12. Description logics
13. CTL and model checking
14. Modal first-order logics

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Organization

Lectures: Where, When, Web Page

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Where

Bld. 101 SR 00-010/014

When

Monday: 12:15–14:00

Wednesday: 12:15–13:00 (+ exercises: 13:15–14:00)

Summer Term Holidays

May 23, 2015 – May 31, 2015

Web Page

`http://www.informatik.uni-freiburg.de/~ki/teaching/ss15/ml/`

Lecturers

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Dr. Stefan Wölfl

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Consultation: by appointment
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Examination

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Exams

- Written or oral exam in September 2015

Course Prerequisites & Goals

Goals

- Acquiring in-depth knowledge on modal logics and related families of logics, “applications” of modal logics in AI, proof techniques, relationship between expressiveness and complexity of logical formalisms, . . .
- Understanding the principles behind techniques for solving reasoning problems in modal logics
- Being able to read and understand research literature
- Being able to complete a thesis in this research area

Prerequisites

- Basic knowledge in the area of AI
- Basic knowledge in formal logic
- Basic knowledge in theoretical computer science

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Literature

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Patrick Blackburn, Maarten de Rijke, and Yde Venema.
Modal Logic,
Cambridge University Press, 2002

M.J. Cresswell and G.E. Hughes.
A New Introduction to Modal Logic,
Routledge, 1996

Further readings will be given during the lecture.