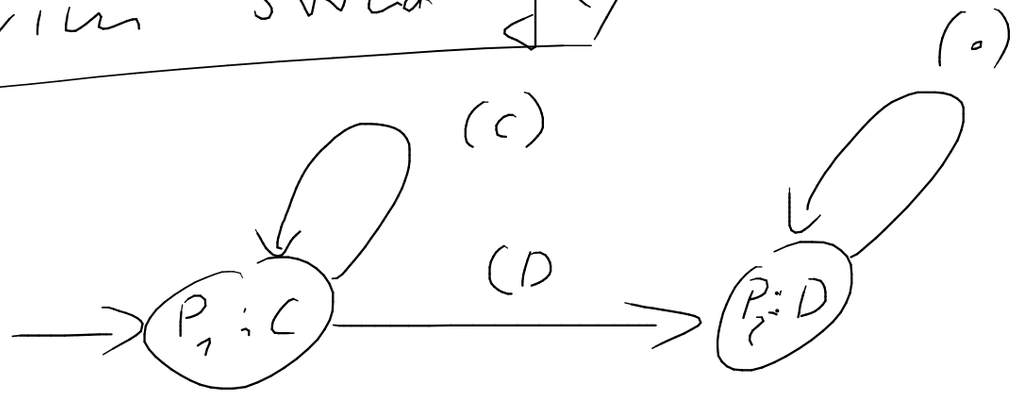
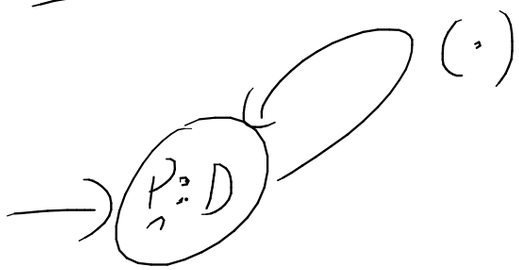
Grim strategy



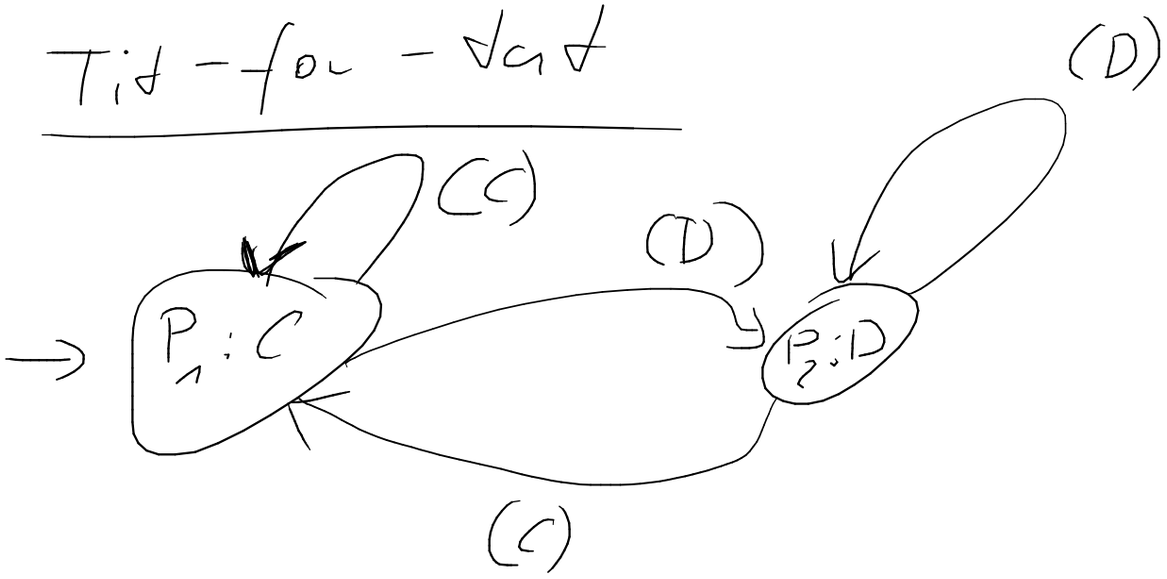
g

Defecting strategy



d

Tit-for-tat



t

Is the grim strategy a NE?

$$V_i(O(g,g)) = 3 + 3 \cdot \frac{\delta}{1-\delta} = 3 \cdot (1 + \delta + \delta^2 + \dots)$$

$$O(g,g) = \langle (C,C), (C,C), \dots \rangle$$

$$\sum_{t=1}^{\infty} \delta^t = \delta + \delta^2 + \dots$$

g' - choose D at some point once

$$\langle (C,C), (C,C), \dots | (C,D), (D,\cdot), (D,\cdot), (D,\cdot), \dots \rangle$$

player 2 has to play D to get the most out of it

g'' - choose D at some point and then ever after

g'' against g is better than g' against g

$$v_2(O(g, g'')) = ?$$

Let's look at $g''' = d$

$$v_2(O(g, d)) = 4 + 1\cdot\delta + 1\cdot\delta^2 + \dots$$

$$\underline{v_2(O(g, g))} = 3 + 3\cdot\delta + 3\cdot\delta^2 + \dots$$

$$\boxed{4 + \frac{\delta}{1-\delta} \leq 3 + \frac{3\delta}{1-\delta}}$$

if this is satisfied then there is no incentive to change from g to d

$$1 + \frac{\delta}{1-\delta} \leq \frac{3\delta}{1-\delta}$$

$$1 \leq \frac{2\delta}{1-\delta}$$

$$1 - \delta \leq 2\delta$$

$$1 \leq 3\delta$$

$$\frac{1}{3} \leq \delta$$

if $\delta \geq 1/3$, g is at least as good as d

This means (g, g) is a NE!
[if δ is large enough]

What about (d, d)?

$\rightarrow$ is also a NE!

What about (t, t)

$\rightarrow$ is also a NE!

Positive message: In repeated games, there are other NEs than the (D, D)

Negative message: Which one do play?

Social Choice Theory

- Aggregation of preferences of group members
- Voting and voting protocols
  - Elections
  - Committee decisions
  - European Song Contest

Def (Social welfare functions and social choice functions)

Let A be a finite set of alternatives (candidates) and L be the set of linear orders over A. For n voters $F: L^n \to L$ denotes a <u>social welfare function</u> and $f: L^n \to A$ a <u>social choice function</u>.

<u>Notation</u>: A linear order $\prec \in L$ is called a <u>preference relation</u>. For a voter i, $\prec_i$ is the preference relation for i. For example $a \prec_i b$ means voter i prefers b over a.

Example:

Three voters: 1, 2, 3

Candidates: a, b, c

	1	2	3
	a	b	c
	b	c	a
	c	a	b

$b <_1 a$, $c <_1 a$, $c <_1 b$

$c <_2 b$, $a <_2 b$, ...

If we have many voters, a table can specify how many voters agree on one list

#votes	8	2	1	3
	a	b	c	a
	b	c	a	c
	c	a	b	b

← number of voters

Voting protocols

1) What the voter has to submit (and when).
2) How to compute the result.

- Plurality (aka "first-past-the-goal", "winner-takes-it-all")
  - Voters submit only their top preference.
  - candidate with most votes wins

  Drawback: Winner might not have more than 50%
- Plurality with runoff
  - Voters submit top preference
  - The two candidates with most go to second round
  - In the second voters decide between the two candidates.

- Instant runoff voting (transferable votes)
  - each voter gives full preference list
  - iteratively candidates with the least number of top preferences are eliminated until only one candidate remains.
- Borda count
  - each voter submits his preference order
  - if a candidate is in position j of a voter's list, he gets m-j points from that voter
  - points are added up
  - the one with the most points wins

3 Condorcet winner

- each voter submits his preferences
- perform pairwise comparisons between candidate (how many voter prefer the one over the other).
- if one candidate wins all the comparisons, this is the winner

<u>Drawback</u>: This winner might not exist because the resulting order is cyclic.

In general, all methods might come up with draws, but this is usually ignored because in large elections it is very unlikely.

Running Example:

23 voters, candidates: a, b, c, d, e

#votes	8	6	4	3	1	1
1st	e	a	b	c	d	d
2nd	d	b	c	b	c	c
3rd	b	c	d	d	a	b
4th	c	e	a	a	b	e
5th	a	d	e	e	e	a

Running Example:

23 voters, candidates: a, b, c, d, e

# votes	8	6	4	3	1	1
1st	e	a	b	c	d	d
2nd	d	b	c	b	c	c
3rd	b	c	d	d	a	b
4th	c	e	a	a	b	e
5th	a	d	e	e	e	a

Plurality: e wins, because e has 8 votes (out of 23)

Running Example:

23 voters, candidates: a, b, c, d, e

#votes	8	6	4	3	1	1
1st	e	a	~~b~~	~~c~~	~~d~~	~~d~~
2nd	~~d~~	~~b~~	~~c~~	~~b~~	~~c~~	~~c~~
3rd	~~b~~	~~c~~	~~d~~	~~d~~	a	~~b~~
4th	~~c~~	e	a	a	~~b~~	e
5th	a	~~d~~	e	e	e	a

8x $a \prec e$

6x $e \prec a$

4x $e \prec a$

3x $e \prec a$

1x $e \prec a$

1x $a \prec e$

9x e is winner

14x a is winner

Plurality with runoff

1st round: e and a

2nd round: 9x e wins 14x a wins

→ a is the winner

Running Example:

23 voters, candidates: a, b, c, d, e

#votes	8	6	4	3	1	1
1st	e	~~a~~	~~b~~	c	~~d~~	~~d~~
2nd	~~d~~	~~b~~	c	~~b~~	c	c
3rd	~~b~~	c	~~d~~	~~d~~	~~a~~	~~b~~
4th	c	e	~~a~~	~~a~~	~~b~~	e
5th	~~a~~	~~d~~	e	e	e	a

Instant runoff

1st round: eliminate d

2nd round: eliminate b

3rd round: eliminate a

Now e has 15 versus 8 for e!

$\longrightarrow$ <u>e wins</u>

Running Example:

23 voters, candidates: a, b, c, d, e

#votes	8	6	4	3	1	1	
1st	e	a	b	c	d	d	4
2nd	d	b	c	b	c	c	3
3rd	b	c	d	d	a	b	2
4th	c	e	a	a	b	e	1
5th	a	d	e	e	e	a	0

Borda count

a: $8 \times 0 + 6 \times 4 + 4 \times 1 + 3 \times 1 + 1 \times 2 + 1 \times 0 = 33$

b: $= 62$

c: $= 50$

d: $= 46$

e: $= 39$

$\rightarrow$ b wins

Running Example:

23 voters, candidates: a, b, c, d, e

#votes	8	6	4	3	1	1
1st	e	a	b	c	d	d
2nd	d	b	c	b	c	c
3rd	b	c	d	d	a	b
4th	c	e	a	a	b	e
5th	a	d	e	e	e	a

$\prec$	a	b	c	d	e
a	—	16	17	17	9
b	7 ($b \prec a$)	—	5	10	8
c	6	18	—	10	8
d	6	13	13	—	14
e	14	15	15	9	—

in all pairwise comparisons b wins

→ b is a Condorcet

- Different winners for different voting protocols!

- Which one should be used?

→ Choice of the voting protocol can be used strategically

Schulze method

→ Condorcet method

→ has a technique for breaking

→ Ubuntu, Pirate party, others use it
