Repeated Games

- If a game is played repeatedly, with the same players, players might behave differently than in one-shot games.
  - finitely repeated: you play the game for a known number of rounds
  - infinitely repeated: infinite number of rounds.
  - indefinitely repeated: you have given probability p that the current game is the last one.

Def.

Let $G = \langle N, (A_i), (u_i) \rangle$ be a strategic game;
let $A = \prod_{i \in N} A_i$. A repeated game with $k \in \mathbb{N} \cup \{\infty\}$ moves is an extensive game with perfect information and simultaneous moves $\Gamma = \langle N, A, H, P, (v_i) \rangle$ with

$$H = \{\emptyset\} \cup \left(\bigcup_{t=1}^{k} A^t \right) \cup A^k$$

$P(h) = N$ for each nonterminal history $h \in H$

If $k \in \mathbb{N}$:

$$v_i(h) = \sum_{t=1}^{k} u_i(a^t) \quad \text{for } a^1 a^2 \ldots a^k = h \quad \text{for all terminal histories } h$$

If $k = \infty$:

Let $\delta \in]0,1[$ be given

$$v_i(h) = \sum_{t=1}^{\infty} \delta^t \cdot u_i(a^t) \qquad a^1 a^2 \ldots = h$$

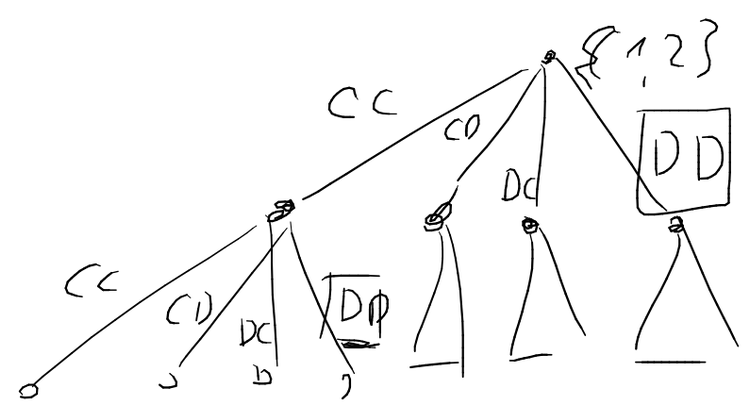
Example: $\delta = \frac{1}{2}$ $\quad u_i(a^t) = 1$

$$\sum_{t=1}^{\infty} \delta^t \cdot u_i(a^t) = \sum \left(\frac{1}{2}\right)^t \cdot 1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots$$

Finite Repetition of a Game with Unique Equilibrium

		D confess	C don't confess
D	confess	1, 1	4, 0
C	don't confess	0, 4	3, 3

- Playing 2 rounds of PD, will we have a different NE?



- Playing this 100 rounds, DD always is the SPE.

→ For finite repetitions, we always have the same unique equilibrium.

Infinitely repeated games

- Finitely repeated games are rare and you do not interesting results.

- Discounting: what is the $\delta + \delta^2 + \delta^3 \ldots = ?$
it converges to $\frac{\delta}{1-\delta}$ for all values $0 < \delta < 1$.

<u>Proof</u> $x = \delta + \underbrace{\delta^2 + \delta^3 + \ldots}$

$$x = \delta + \delta(\underbrace{\delta + \delta^2 + \delta^3 + \ldots})$$

$$x = \delta + \delta x$$

$$x - \delta x = \delta$$

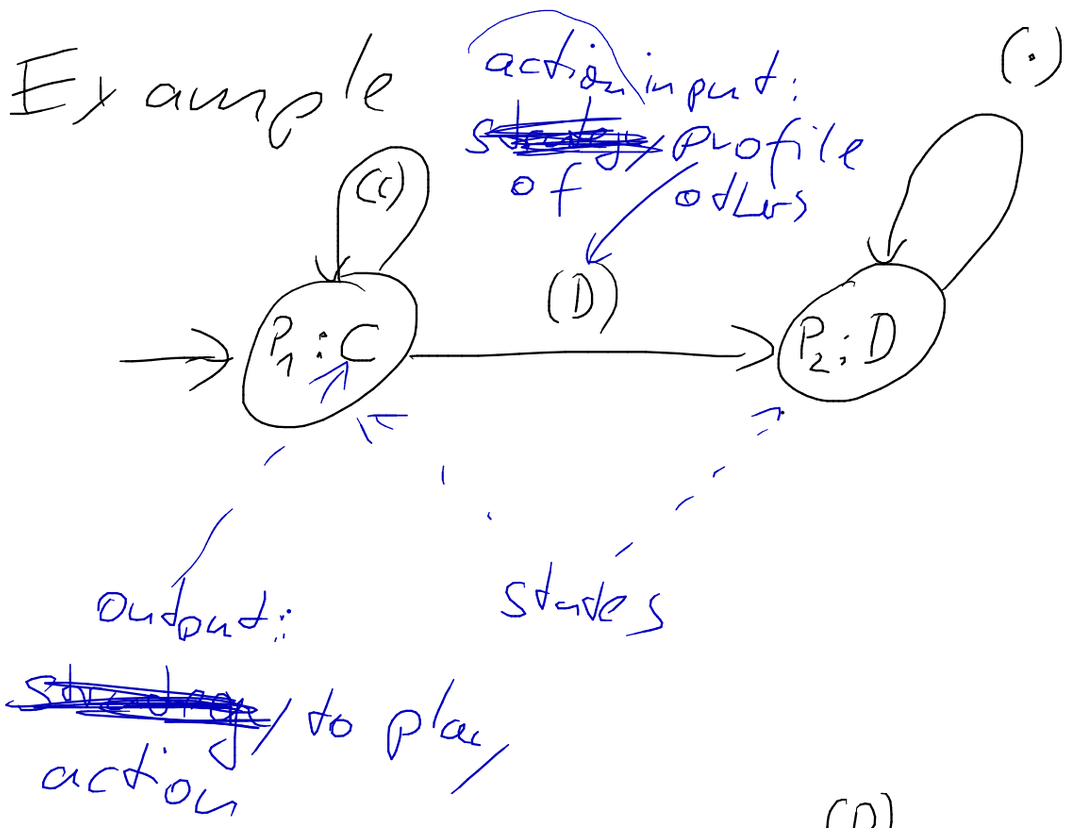
$$x(1-\delta) = \delta$$

$$\boxed{x = \frac{\delta}{1-\delta}}$$

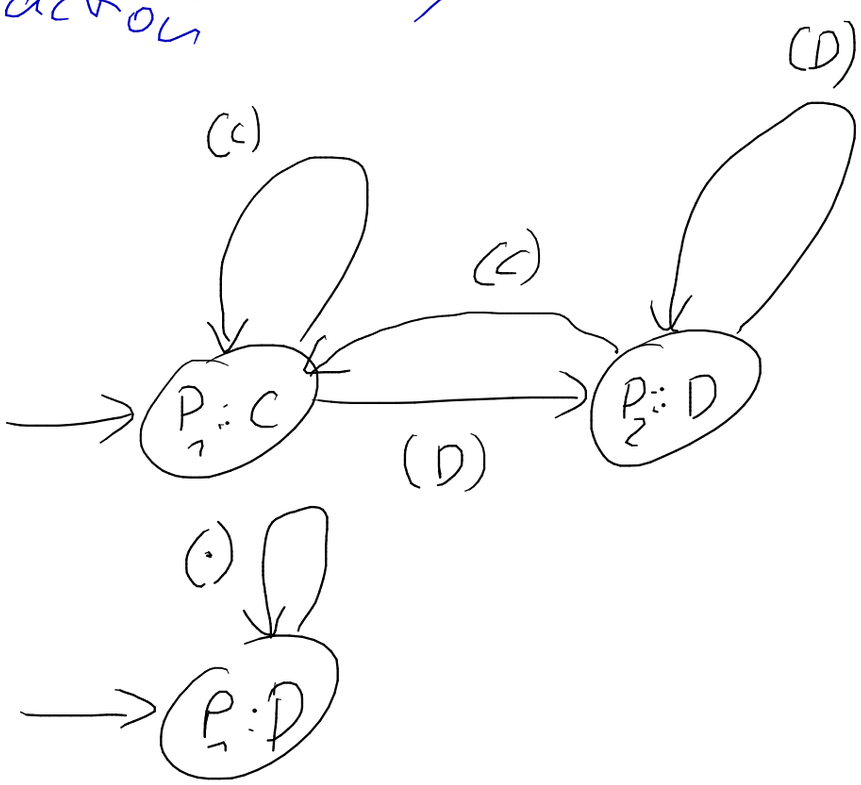
Strategies in infinite games:

Finite automatons (Moore)

Example



Grim strategy g



Tit-for-Tat strategy t

Defecting strategy d

$V_i(g, g) = \frac{\delta}{1-\delta} \cdot 3$ is the overall utility of the repeated game