

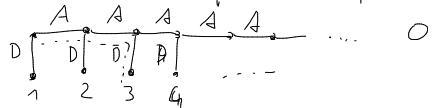
## Theorem (Kuhn)

Every finite extensive game with perfect information has an SPE.

Proof idea:

- Backward induction.

Counterexample (infinite horizon)



1

Backward induction can be seen as a generalization of the minimax procedure for solving board games.

Assumption: 2 players called MIN and MAX.

We can evaluate terminal board positions (checkmate, for example) in order to get utility values,

For example, utility values for MAX:

1 - MAX has won

0 - draw

-1 - MAX has lost game

For MIN player, the values are just the negative values.

3

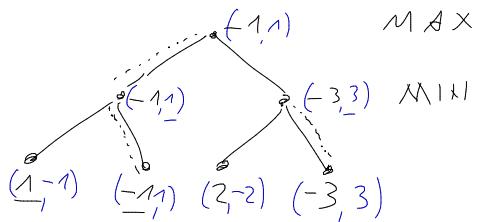
Counterexample (finite horizon, infinite branching)  
One player game with  $A = [0, 1]$ ,  
 $v_1(a) = a$   
There is no best move, i.e., there is no SPE!

2

Minimax:

- Build game tree with MIN and MAX levels (corresponds to player fraction).
- Evaluate the terminal nodes
- Propagate the values from the leaves to the root:
  - For MIN nodes, you choose a child node with minimal value
  - For MAX you choose a child node with maximum value.
  - branch with best value is chosen as next move.

4



Looks similar to backward induction, but only one number?

### -Zero-Sum Game

→ Minimax Procedure Special case of backward induction

5

### Extended Example: Pirate Game

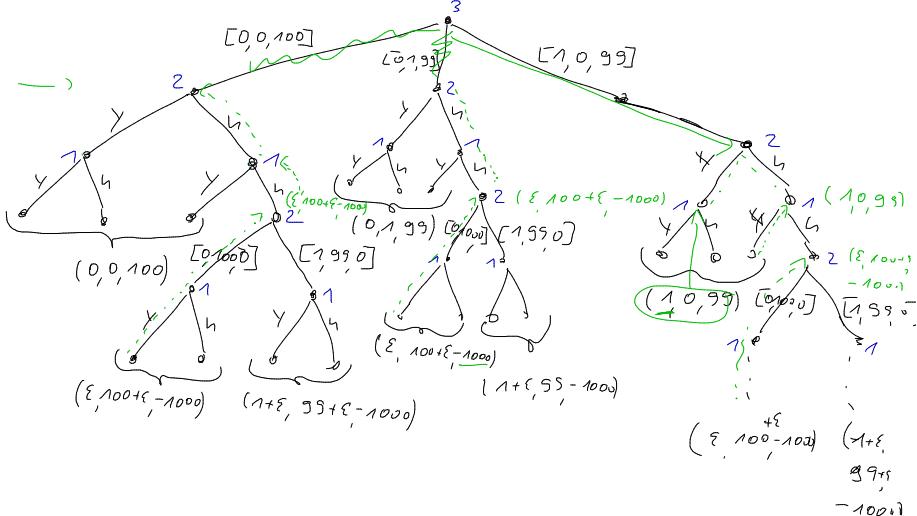
There are  $n \leq 200$  pirates who have found 100 golden coins that have to be distributed among them. The pirates have a strict seniority principle: pirate  $i+1$  is superior to pirate  $i$ .

The most senior pirate proposes a distribution. Then all pirates vote on whether they accept it. In case of a tie, the most senior pirate has a casting vote. If they accept, then the game ends with distributing the money. If not, the proposing pirate is thrown overboard.

First of all, pirates prefer to survive. Secondly, they want to maximize the number of coins. Thirdly, everyone else equal, they prefer to throw other pirates overboard.

6

3 pirates (+ε for each pirate thrown over board, -1000 for being killed)



7

6 Pirates (starting from the bottom):

Pirate 6 is the only one left:  $[100, 0, 0, 0, 0]$  → accepted

Pirates 1 and 2 are alive:  $[0, 100, 0, 0, 0]$  → accepted

Pirates 1, 2, 3 are alive:  $[1, 0, 99, 0, 0]$  → accepted

Pirates 1, 2, 3, 4 are alive:  $[0, 1, 0, 99, 0]$  → accepted

" 1, 2, 3, 4, 5 are alive:  $[1, 0, 1, 0, 98]$  → accepted

" 1, 2, 3, 4, 5, 6 are alive:  $[0, 1, 0, 1, 0, 98]$

⋮

20 pirate

8

What happens with  $> 200$  pirates?

P201: gives all coins to the 100 odd numbered pirates

P202: gives all coins to even numbered pirates

P203: regardless of what he does, he dies

P204: will survive because P203 would die

P205, P206, P207

P208: survives because he gets the votes P205,  
P206 & P207

9

In general: if number of gold coins and  $N (> 26)$  is the number of pirates

- then no pirate with number  $\geq 26$  will get any coins
- all pirates whose number  $\leq 26 + M$  survive, where  $M$  is the highest power of 2 that does not exceed  $N - 26$
- all pirates with a number  $> 26 + M$  will die

10

Def: An EGWPI with simultaneous moves is a tuple  $\Gamma = \langle N, A, H, P, (v_i) \rangle$  where

- $N, A, H$  and  $(v_i)$  are defined as before and
- $P: H \rightarrow 2^N$  assigns to each non-terminal history a set of players. For all  $h \in H \setminus Z$  there exists a family  $(A_i(h))_{i \in P(h)}$  such that
$$A(h) = \{a_i / (h, a_i) \in H\} = \prod_{i \in P(h)} A_i$$

- what about one deviation property?

→ still holds

- what about Kuhn's theorem

→ not necessarily (because there is not necessarily a NE in strategic formes)

11

Def: An EGWPI and chance moves is a tuple  $\Gamma = \langle N, A, H, P, f_c, (v_i) \rangle$ , where

- $N, A, H, v_i$  are the same as before.
- $P: H \setminus Z \rightarrow N \cup \{\text{ch}\}$
- for each  $h \in H \setminus Z$  with  $P(h) = c$ , the function  $f_c(\cdot | h)$  is a probability measure on  $A(h)$  such that the probability measure is independent for all  $h \in H \setminus Z$ .

In intended meaning: with chance move,  $f_c(\cdot | h)$  dictates what the next action to take using an appropriate dice.

The outcome for a given strategy profile

12

- is a distribution over possible histories.
- The resulting utility, is the expected utility  
over all possible outcomes.

---

One deviation property + Kelly's theorem still holds.