One Deviation Property:

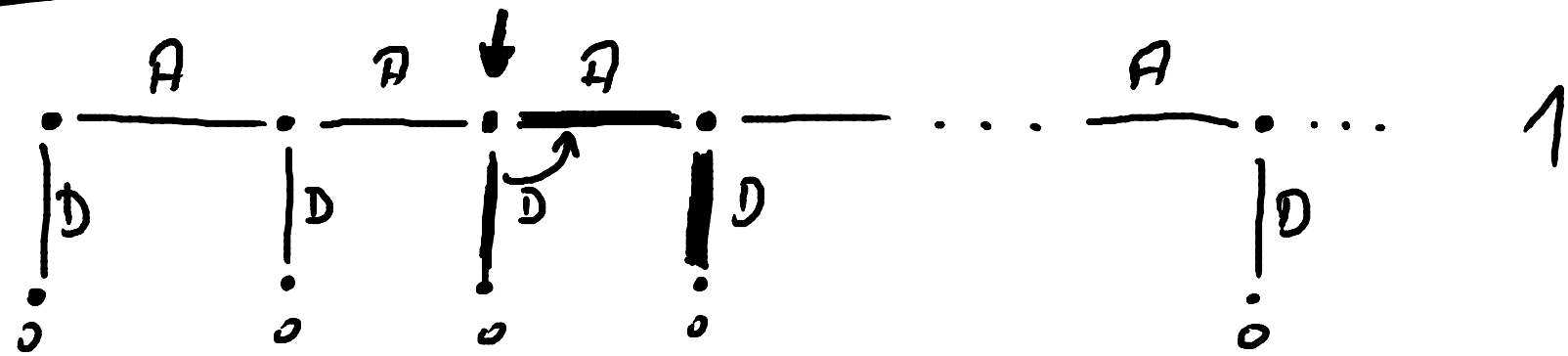
Let $\Gamma = \langle N, A, H, P, (u_i)_{i \in N} \rangle$ be a <u>finite-horizon</u> EGWPI. Then a strategy profile s^* is an SPE of Γ iff for every player $i \in N$ and every history $h \in H$ with $P(h) = i$, we have

$$u_i|_h(O_h(s^*_{-i}|_h, s^*_i|_h)) \geq u_i|_h(O_h(s^*_{-i}|_h, s_i))$$

for every strategy s_i of player i in the subgame $\Gamma(h)$ that <u>differs from $s^*_i|_h$ only in the action after the initial history of $\Gamma(h)$</u>.

In infinite-horizon games, the one-deviation property does <u>not</u> hold.

<u>Counterexample (1-player game)</u>:



strategy $s_1(h) = D$ for all $h \in H \setminus Z$.

Now, s_1 is <u>not</u> a SPE.

For all histories h', there is no profitable deviation from $s_1|_{h'}$ that gives higher payoff than D

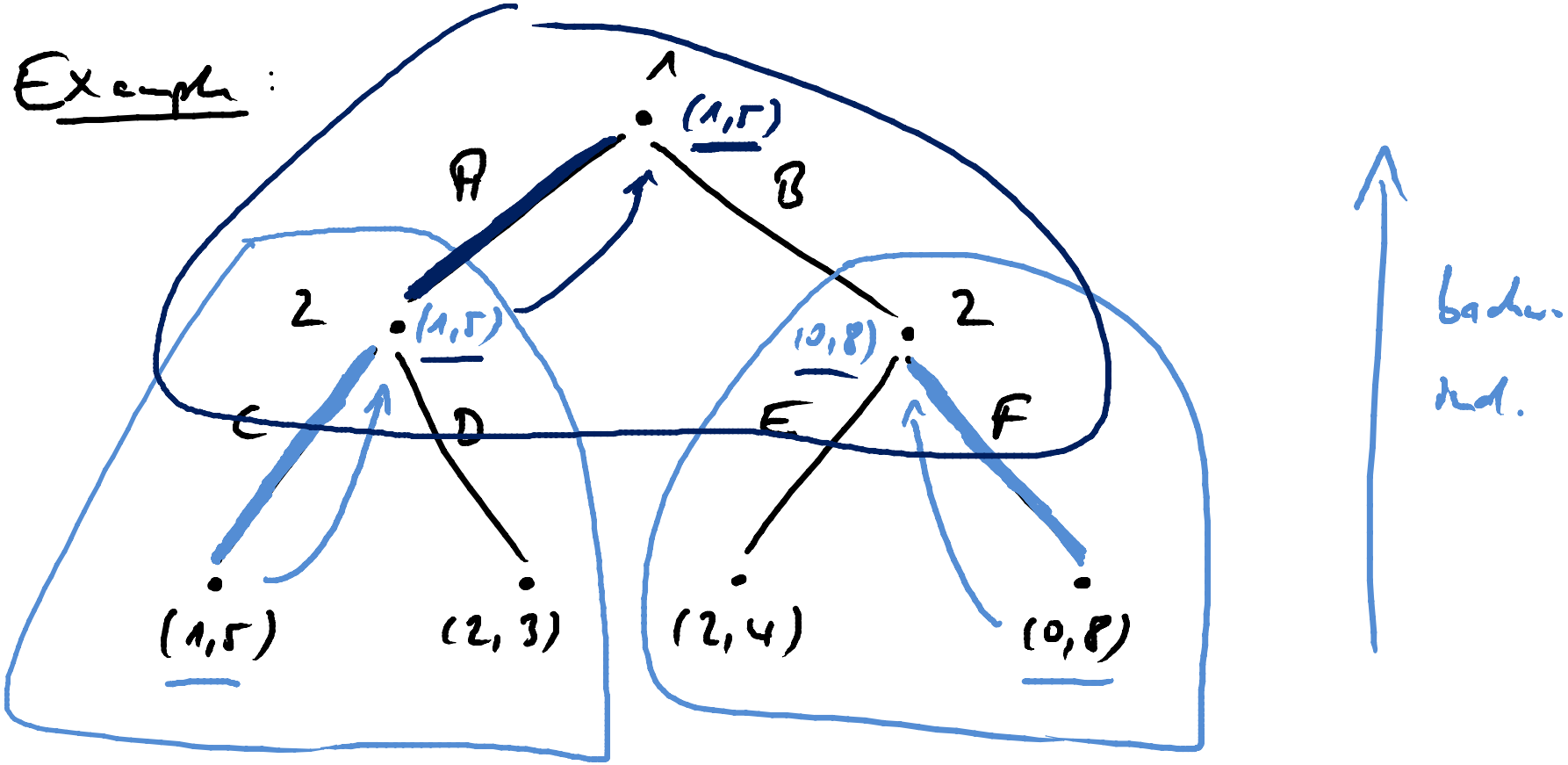
But strategy $s_1^*(h) = A$ f.a. $h \in H \setminus Z$ is a SPE. $(\circlearrowleft)$.

Theorem (Kuhn): Every finite extensive game with perfect information has a SPE.

Proof idea:

* Constructive: build a SPE bottom-up (backward induction)
* Similar to minimax procedure from intro to AI class.

Example:



<u>Proof:</u> Let $\Gamma = \langle N, A, H, P, (u_i) \rangle$ be a <u>finite</u> EGUPI. Construct a SPE by induction on $\ell(\Gamma(h))$ f.a. subgames $\Gamma(h)$.

In parallel, construct functions $t_i : H \to \mathbb{R}$ for all players $i \in N$ s.t. $t_i(h)$ is the payoff of player i in a SPE in subgame $\Gamma(h)$.

<u>Base case:</u> If $\ell(\Gamma(h)) = 0$, then $t_i(h) = u_i(h)$ for all $i \in N$.

<u>Inductive case:</u> Assume that $t_i(h)$ is already defined for all $h \in H$ with $\ell(\Gamma(h)) \leq k$. Consider

history $h^* \in H$ with $\ell(\Gamma(h^*)) = k+1$.
Assume $P(h^*) = i$. Let

$$s_i(h^*) := \underset{a \in A(h^*)}{\arg\max}\ t_i(h^*, a),$$

$$t_j(h^*) := t_j(h^*, s_i(h^*))$$

for all $j \in N$.

Inductively, we obtain a strategy profile that satisfies the <u>one-deviation property</u> (by construction). From the 1-dev-prop. lemma it follows that the constructed profile is indeed a SPE. $\square$