Extensive Games

Finite game means the set H is finite.

Finite horizon was defined as "no infinite history" ①

Osborne/Rubinstein: "② If the longest history is finite"

there is a hole in the definition!

all histories are finite

← infinite branching

this is not finite horizon

The "right" definition: "If there exists an upper bound $b \in \mathbb{N}$ for the length of the histories, then the game is a finite horizon game."

Strategies in extensive games

strategies $\neq$ actions

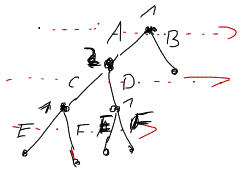
Def (strategy)

Let $\Gamma = \langle N, A, H, P, (u_i) \rangle$ be a EGWPI. Then the set of actions a with $(h,a) \in H$ are denoted by $A(h)$. A strategy of player i is a function s_i that assigns to each non-terminal history $h \in H \setminus Z$ with $P(h) = i$ an action $a \in A(h)$. The set of strategies of player i is denoted by S_i.

Remark: Strategies require us to assign action to histories, even if it is clear they will never be played!

Notation: Strategies are often given by writing the actions going through the game tree in a level-by-level, left-to-right way:

Example:



Strategies for player 1

AEE, AF, BE, BF

Strategies for player 2

C, D

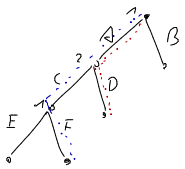
Def (outcome)

The outcome of a strategy profile $s = (s_i)_{i \in N}$ is the history $h^s = (a_k)_{k=1}^{L}$ such that for all $0 \le k \le L$

$L \in \mathbb{N} \cup \{\infty\}$, where $s_{P(a_1, \dots, a_k)}(a_1, \dots, a_k) = a_{k+1}$.

The outcome of strategy s is denoted by $O(s)$.

Example



$O((AF, D)) = (A, D)$

$O((AF, C)) = (A, C, F)$

Def (NE)

A Nash Equilibrium of an extensive game with perfect information Γ is a strategy profile $s^* = (s_i^*)_{i \in N}$ such that for each player $i \in N$:

$$u_i(O(s^*)) \geq u_i(O(s^*_{-i}, s_i)) \text{ for all } s_i \in S_i.$$

Def (game induced by extensive game)

The strategic game G' induced by an extensive game Γ is defined by

$$G' = \langle N, (A_i')_{i \in N}, (u_i') \rangle \quad \text{with}$$

$$A_i' = S_i \qquad A' = S$$

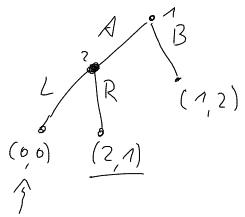
$$u_i'(\underline{a}) = u_i(O(\underline{s})).$$

Proposition

The NE of an EGWPI Γ are exactly the NE of the induced strategic game G'.

Remarks:

1) Each EGWPI can be transformed into a strategic game; but the created game may be exponentially larger.
2) The other direction does not work (because we do not have simultaneous actions).



non-credible threat

	L	R
A	0,0	2,1
B	1,2	1,2

NE, but a very funny one