2.3 Examples – continued

Battle of sexes (Bach or Strawinsky) BoS

Coordination game

Setting: Two people want to go together to a concert. They make an appointment for that, but forgot to choose the concert.
Player 1 likes Bach and player 2 likes Strawinsky.
Both know that they prefer to go to the same concert

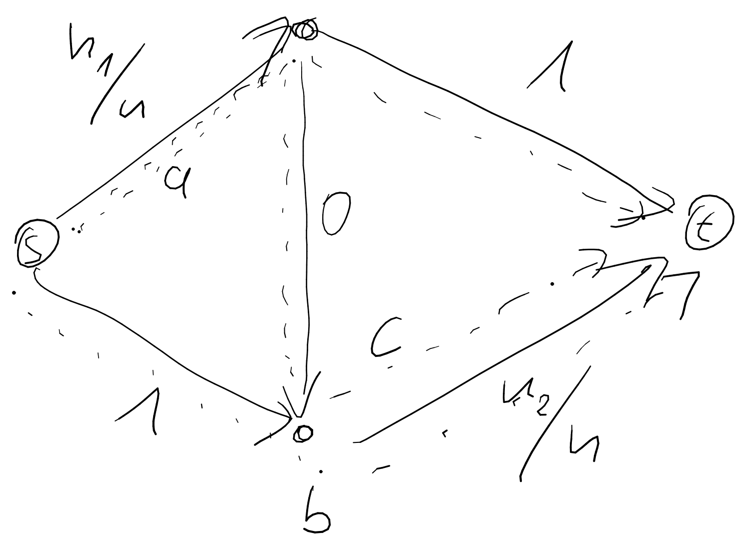
	B	S
B	2,1	0,0
S	0,0	1,2

Dove - Hawk (Chicken game)

Setting: Two agents competing for resources.
They can decide to act like a hawk (attacking) or like a dove (yielding). If both attack, the outcome will be zero for both. If one attacks and one yields, the attacker has a high payoff, the yielder has a small non-zero payoff. If both yield, then they get slightly higher payoff than the dove in the former situation.

	H	D
H	0,0	4,1
D	1,4	2,2

Anti-coordination game



	a	b	c
a	−2, −2	−1.5, −1.5	−2, −1.5
b	−1.5, −1.5	−2, −2	−2, −1.5
c	−1.5, −2	−1.5, −2	−2, −2

Matching Pennies (Penalty Shoots)

<u>Setting</u>: 2 people have to make a choice between "head" and "tails".

If the choices differ, then player 1 gives 1\$ to player 2.

If the choices are the same, then player 2 gives 1\$ to player 1.

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

Example of a <u>zero-sum game</u>

i.e the sums of the utilities for all strategy profiles are all zero. This is also a strictly competitive game.

2.4 Solution Concepts & Notation

- What is a "solution" of a strategic game?

→ strategy profiles where all player play strategies that are rational (i.e. in some sense optimal).

When talking / writing strategy profiles, we want to talk profiles where one player is removed or one action is replaced.

<u>Notation</u> Let $\underline{a = (a_1, \dots, a_{|N|})} \in \prod_{i \in N} A_i := A$

$A_{-i} := \prod_{j \in N \setminus \{i\}} A_j$ $\qquad a_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_{|N|})$

Given a strategy profile $a = (a_1, \dots, a_{i-1}, a_i, a_{i+1}, \dots, a_{|N|})$

let $a_i' \in A_i$ then $(a_{-i}, a_i') := (a_1, \dots, a_{i-1}, a_i', a_{i+1}, \dots, a_{|N|})$

Example

$A_1 = \{T, B\}$ $\quad A_2 = \{L, R\}$ $\quad A_3 = \{X, Y, Z\}$

$a := (T, R, Z)$ $\quad a_{-2} = (T, Z)$ $\quad a_{-3} = (T, R)$

$(a_{-2}, L) = (T, L, Z)$

2.5 Elimination of Dominated Strategies

What strategy should an agent avoid?

→ Eliminate all irrational strategies!

Def A strategy $a_i' \in A_i$ is strictly dominated by a strategy $a_i^+ \in A_i$ if for all profiles $a \in A$:

$$\boxed{u_i(a_{-i}, a_i') < u_i(a_{-i}, a_i^+)}$$

Remark: Playing strictly dominated strategies is irrational.

Examples

	Confess	Don't Confess
Confess	1,1	4,0
Don't Confess	0,4	3,3

	B	S
B	2,1	0,0
S	0,0	1,2

Def A strategy $a_i' \in A_i$ is called <u>weakly dominated</u> by $a_i^+ \in A_i$ if for each $a \in A$:

$$u_i(a_{-i}, a_i') \leq u_i(a_{-i}, a_i^+)$$

and if there exist at least one profile $a \in A$ such that

$$u_i(a_{-i}, a_i') < u_i(a_{-i}, a_i^+).$$

	L	R
T	2,1	0,0
M	2,1	1,1
B	0,0	1,1

$\rightarrow$ iterated elimination of dominated strategies (IEDS)

$u_1(M, L) = 2 > u_1(B, L) = 0$

$u_1(M, R) = 1 \geq u_1(B, R) = 1$

M weakly dominates B, B should be eliminated!

Assuming that all agents eliminate dominated strategies, this process can be iterated!

→ This leads to smaller games

→ This also can lead to the situation, where only one profile remains.

→ <u>Note</u>: With elimination of weakly dominated strategies, the result can depend on the order of the elimination.

In our example:

$T \to L \to M$ or B, R

$B \to R \to T$ or M, L

<u>Example</u>: Beauty contest with 2 players, maximum = 4, and rounding down.

The winner is the one closest to 2/3 of the average rounded to down to the next integer

$\lfloor 2/3 \, \emptyset \rfloor$

	0	1	2	3	4
0	1,1	1,0	1,0	1,0	1,0
1	0,1	1,1	1,0	1,0	1,0
2	0,1	0,1	1,1	1,0	1,0
3	0,1	0,1	0,1	1,1	1,0
4	0,1	0,1	0,1	0,1	1,1

weakly dominating strategy

	0	1	2	3	4
0	1,1	1,1	1,1	1,0	1,1
1	1,1	1,1	1,0	1,1	1,0
2	1,1	0,1	1,1	1,0	1,0
3	0,1	1,1	0,1	1,1	1,0
4	1,1	0,1	0,1	0,1	1,1