

## 2. Strategic Games

- a finite set of players
- each player has a set of actions / strategies
- the outcome (utility) for each player depends on the actions of all players (= action profile)

### Beauty contest

Every body chooses a number  $n$  with  $0 \leq n \leq 100$ . We consider the average of all numbers. The agent that comes closest to  $\frac{2}{3}$  of the average (rounded up) wins ... and gets a US \$ 1000000.

## 2.1 Defining Games

Def. A strategic game is a triple

$$G = \langle N, (A_i)_{i \in N}, (v_i)_{i \in N} \rangle \text{ with}$$

- (a)  $N$  is a finite set (of agents) players
- (b) for player  $i \in N$ , a non empty set  $A_i$  (set of actions / strategies),  
 $A = \prod_{i \in N} A_i$  is called set of action profiles
- (c) for each player  $i \in N$ , a utility function:  
 $v_i : A \rightarrow \mathbb{R}^+$

Finite game  
has only finite  
sets of actions

## 2.2 Payoff matrix

For 2 players and finite games G we can represent strategic games by payoff matrices, e.g.  
for  $N = \{1, 2\}$ ,  $A_1 = \{\overline{T}, \overline{B}\}$ ,  $A_2 = \{\overline{L}, \overline{R}\}$ ,

		$L$	$R$	
		$w_1, w_2$	$x_1, x_2$	
$\text{Player}_1$	$\overline{T}$	$w_1, w_2$	$x_1, x_2$	$w_1, w_2, x_1, \dots, z_2 \in \mathbb{R}^+$
	$\overline{B}$	$y_1, y_2$	$z_1, z_2$	

Profiles  $(T, L)$        $(T, R)$        $(B, L)$        $(B, R)$

$\downarrow$

$(w_1, w_2)$

$v_1(T, L) = w_1$      $v_2(T, L) = w_2$

21-2  $\mapsto$  22 is the winner

## 2.2 Example games

### Prisoner's dilemma:

Two suspects in crime are brought for questioning in different cells:

- \* If both confess, then both will receive 3 year sentence in prison
- \* If neither of them confess, they only go into prison for 1 year.
- \* If one confesses and they does not, the first one walks free, the second one goes in to prison for 4 years

	Confess	Don't Confess
Confess	-3, -3	0, -4
Don't Confess	-4, 0	-1, -1

and el + y

+ b everything

	Confess	Don't Confess
Confess	1, 1	4, 0
Don't Confess	0, 4	3, 3