Introduction to Game Theory

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Summer semester 2015

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Exercise Sheet 11

Due: Friday, July 17th, 2015

Exercise 11.1 (May's theorem, 4 points)

Recall May's theorem: A social choice function $f : L^n \to A$ for a set of two alternatives $A = \{x, y\}$ satisfies anonymity, neutrality and monotonicity iff it is the plurality method (i.e., $f(\prec_1, \ldots, \prec_n) = x$ iff $\#\{i \mid y \prec_i x\} \geq \frac{n}{2}$).

We assume n is odd to avoid tie-breaking issues that could violate neutrality.

Show that each of the three conditions is necessary for May's theorem.

(a) anonymity, i.e., $f(\prec_1, \ldots, \prec_n) = f(\prec_{\pi(1)}, \ldots, \prec_{\pi(n)})$ for all permutations π of the voters $\{1, \ldots, n\}$.

(b) neutrality, i.e., $f(\prec_1, \ldots, \prec_n) = x$ iff $f(\prec'_1, \ldots, \prec'_n) = y$, where $x \prec'_i y$ iff $y \prec_i x$ for all $i = 1, \ldots, n$.

(c) monotonicity, i.e., if $f(\prec_1, \ldots, \prec_n) = x$, then also $f(\prec'_1, \ldots, \prec'_n) = x$, where $\prec'_i = \prec_i$ for $i \neq I$ for some voter I such that $x \prec_I y$ and $y \prec'_I x$.

Hint: For each condition, find a counterexample (a social choice function) that fulfills all other conditions but the one in question and that is not the plurality method.

Exercise 11.2 (Schulze method, 4 points)

For the following preference relations determine the set of possible winners according to the Schulze-method[1]:

20 voters have the preference $b \prec_i c \prec_i e \prec_i d \prec_i a$
10 voters have the preference $d \prec_i e \prec_i c \prec_i b \prec_i a$
15 voters have the preference $b \prec_i d \prec_i a \prec_i e \prec_i c$
12 voters have the preference $a \prec_i b \prec_i c \prec_i e \prec_i d$
13 voters have the preference $a \prec_i e \prec_i c \prec_i d \prec_i b$

The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.

[1] `http://en.wikipedia.org/wiki/Schulze_method`