Introduction to Game Theory

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Exercise Sheet 10

Due: Friday, July 10th, 2015

Exercise 10.1 (Voting procedures, 4 points)

Consider the following voting procedures (for simplicity, we assume that ties are broken in favor of the candidate with the lower index):

Plurality vote: Only top preferences are taken into account. The candidate with most top preferences wins.

Instant runoff voting: Iteratively candidates with the fewest top preferences are eliminated until only one candidate, the winner, remains.

Coombs method: Iteratively candidates with the most bottom (lowest) preferences are eliminated until only one candidate, the winner, remains.

Borda count: If a candidate is in position j of a voter's preference list, he gets $m - j$ points from that voter. Points from all voters are added. The candidate with most points wins.

Give preference relations $\prec_1, \dots, \prec_n$ over a candidate set $A = \{a_1, \dots, a_m\}$ such that the above-mentioned voting procedures return as many different winners as possible. You will obtain one point per different winner.

Exercise 10.2 (Properties of voting procedures, 4 points)

Consider the voting procedures **plurality vote**, **instant runoff voting**, and the **Borda count**. Again, we assume that ties are broken in favor of the candidate with the lower index. Moreover, $|A| \geq 3$. Consider the following properties:

Majority criterion: If for more than half of the voters i, $b \prec_i a$ for all $b \in A \setminus \{a\}$, then $f(\prec_1, \dots, \prec_n) = a$.

Reversal symmetry: If $f(\prec_1, \dots, \prec_n) = a$ and $a \prec'_i b$ iff $b \prec_i a$ for all $i = 1, \dots, n$ and $a, b \in A$, then $f(\prec'_1, \dots, \prec'_n) \neq a$.

Incentive compatibility: $f(\prec_1, \dots, \prec'_i, \dots, \prec_n) \preceq_i f(\prec_1, \dots, \prec_i, \dots, \prec_n)$ for all $\prec_1, \dots, \prec_n, \prec'_i \in L$.

For each of the nine combinations of voting procedure f and property P, show that f satisfies P or give a counterexample.

The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.