

Introduction to Game Theory

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Exercise Sheet 5

Due: Monday, June 8th, 2015

Exercise 5.1 (Linear Complementary Problem, 3 + 1 points)

Consider the strategic game $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$, with

- $N = \{1, 2\}$,
- $A_1 = A_2 = \{r_1, r_2, r_3\}$ and
- utility functions u_1, u_2 as given by the following payoff matrix.

		Player 2		
		r_1	r_2	r_3
Player 1	r_1	0, 0	3, 1	3, 3
	r_2	1, 1	0, 0	1, 3
	r_3	1, 1	1, 1	0, 0

- (a) Determine the mixed strategy Nash equilibria for this game. Proceed as follows:
1. Formulate the corresponding LCP.
 2. Convert the LCP into a linear program with the following pair of support sets: $(\text{supp}(\alpha), \text{supp}(\beta)) = (\{r_1, r_2, r_3\}, \{r_1, r_2, r_3\})$.
 3. Solve the linear program and provide values for each $\alpha(r_i)$ and $\beta(r_i)$, $i \in \{1, 2, 3\}$.
- (b) What is the expected payoff (u, v) of the NE computed above?

Exercise 5.2 (Penalty-Kicks, 2 + 2 points)

The following two games depict a penalty-kick scenario, where player 1 (Kicker) can shoot and player 2 (Goalie) jump either to the left (L) or right (R) side of the goal. In the first game the Goalie catches the ball only if he jumps to the same side the Kicker is shooting at. In the second game the Kicker sometimes misses the goal when shooting to the right side, which is reflected in the payoffs.

		Goalie	
		L	R
Kicker	L	0, 1	1, 0
	R	1, 0	0, 1

		Goalie	
		L	R
Kicker	L	0, 1	1, 0
	R	$\frac{3}{4}, \frac{1}{4}$	0, 1

- (a) Compute the mixed strategy Nash equilibria for both games.
- (b) The results look counter-intuitive when comparing the Kickers equilibrium strategy of the first game to the one of the second game. How are they different and how can the difference be explained?