Introduction to Game Theory

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Exercise Sheet 3

Due: Friday, May 15, 2015

Exercise 3.1 (Best response function, 3 points)

Let $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$ with $N = \{1, 2\}$, $A_1 = A_2 = \mathbb{R}^{\geq 0}$, $u_1(a_1, a_2) = a_1(a_2 - a_1)$ and $u_2(a_1, a_2) = a_2(1 - \frac{1}{2}a_1 - a_2)$ for all $(a_1, a_2) \in A$.
Define all Nash equilibira of this game by constructing and analyzing the best response function of both players.

Exercise 3.2 (Minimax strategy profiles, 1.5+1.5 points)

Let G be a zero-sum game that has a Nash equilibrium.

(a) Show that if some of player 1's payoffs are increased in such a way that the resulting game G' is also a zero-sum game then G' has no Nash equilibrium in which player 1 gets a lower payoff than he got in the Nash equilibria of G.

(b) Show that the game G' that results from G by elimination of one of player 1's strategies does not have a Nash equilibrium in which player 1's payoff is higher than it is in the Nash equilibria of G.

Exercise 3.3 (Nash equilibria in zero-sum games, 2 points)

Prove the following claim or give a counterexample: If G is a zero-sum game that has a Nash equilibrium with payoff v for player 1 then every strategy profile in G with payoff v for player 1 is a Nash equilibrium.

The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.