Foundations of Artificial Intelligence

Prof. Dr. W. Burgard, Prof. Dr. B. Nebel, Prof. Dr. M. Riedmiller
J. Aldinger, J. Boedecker, P. Ruchti
University of Freiburg
Department of Computer Science
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Exercise Sheet 5

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Exercise 5.1 (Predicate Logic)
Consider following colloquial sentences:

(a) Not all students attend AI and ST.

(b) One student failed both AI and ST.

(c) Exactly two students failed ST.

(d) There is a barber who shaves all men in town who do not shave themselves.

(e) No one likes a professor who is not smart.

Represent these sentences in first-order logic using the predicates *student(x)*, *attends(x,y)*, *fails(x,y)*, *barber(x)*, *shaves(x,y)*, *professor(x)*, *likes(x,y)* und *smart(x)*.

Exercise 5.2 (Semantics of Predicate Logic)
Consider the Interpretation $\mathcal{I} = <\mathcal{D}, \cdot^{\mathcal{I}}>$ with

- $D = \{0, 1, 2, 3\}$
- $even^{\mathcal{I}} = \{0, 2\}$
- $odd^{\mathcal{I}} = \{1, 3\}$
- $lessThan^{\mathcal{I}} = \{(0,1), (0,2), (0,3), (1,2), (1,3), (2,3)\}$
- $two^{\mathcal{I}} = 2$
- $plus^{\mathcal{I}} : D \times D \to D, plus^{\mathcal{I}}(a, b) = (a + b) \mod 4$

and the variable assignment $\alpha = \{(x, 0), (y, 1)\}$.
Decide for the following formulae θ_i if $\mathcal{I}$ is a model for θ_i under α, i.e. if $\mathcal{I}, \alpha \models \theta_i$.
Explain your answer.

(a) $\theta_1 = odd(y) \wedge even(two)$

(b) $\theta_2 = \forall x\ (even(x) \vee odd(x))$

(c) $\theta_3 = \forall x \exists y\ lessThan(x, y)$

(d) $\theta_4 = \forall x\ (even(x) \Rightarrow \exists y\ lessThan(x, y))$

(e) $\theta_5 = \forall x\ (odd(x) \Rightarrow even(plus(x, y)))$

Exercise 5.3

(a) Transform the following formula into Skolem Normal Form (SNF):

$$\forall z \exists y (P(x, g(y), z) \vee \neg \forall x Q(x)) \wedge \neg \forall z \exists x \forall t \neg R(f(x, z), z, t)$$

(b) Give the 10 smallest terms in the Herbrand universe and the 10 smallest formulae in the Herbrand expansion of the following formula:

$$\forall x \forall y P(c, f(x, b), g(y))$$

Exercise 5.4 (Substitutions and Unification)

(a) Compute the substitutions

(i) $P(x, y)\{\frac{x}{A}, \frac{y}{f(B)}\}$,
(ii) $P(x, y)\{\frac{x}{f(y)}\}\{\frac{y}{g(B,B)}\}$,
(iii) $P(x, y)\{\frac{x}{f(y)}, \frac{y}{g(B,B)}\}$ and
(iv) $P(x, y)\{\frac{z}{f(B)}, \frac{x}{A}\}$

(b) Apply the unification algorithm to the following set of literals:

$$\{R(h(x), f(h(u), y)), R(y, f(y, h(g(A))))\}$$

In each step, give the values of T_k, s_k, D_k, v_k, and t_k.

Exercise 5.5 (Allen's Interval Calculus)

(a) Consider the non-empty intervals *Match*, *GoalShot*, *Cheering* und *FinalWhistle* together with the constraints

(i) *FinalWhistle f Match*
(ii) *GoalShot m Cheering*
(iii) *GoalShot (d,f) Match*
(iv) *GoalShot (<,m) FinalWhistle*

Which of the following relations are entailed?

(a) *GoalShot d Match*
(b) *Cheering d Match*

(b) In general, the composition of two binary relations R and S (over X) is defined as

$$R \circ S = \{(x, z) \mid \exists y \in X \text{ such that } (x, y) \in R \text{ and } (y, z) \in S\}.$$

Allen's interval calculus is *closed under composition* which means that every composition of Allen relations (also for unions of the 13 base relations) can be represented as union of base relations. For example, $f \circ s = d$ because for arbitrary intervals A, B and C with AfB and BsC it must hold that AdC. Note that in general the composition of two base relations needs not to result in a single base relation, as you can see from the example $f^{-1} \circ d = (o, d, s)$. Determine the following compositions:

(1) $o \circ m$

(2) $m \circ f$

(3) $(o, f^{-1}) \circ f$

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.