Foundations of Artificial Intelligence

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Exercise Sheet 4

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Exercise 4.1 (Satisfiability, Models)

(a) Decide for each of the following propositions whether they are valid, satisfiable or neither valid nor satisfiable.

(i) $Smoke \Rightarrow Smoke$
(ii) $Smoke \Rightarrow Fire$
(iii) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Fire \Rightarrow \neg Smoke)$
(iv) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$
(v) $TheBestTeamWins \Leftrightarrow GermanyWinsWorldCup$

(b) Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following formulae? Explain.

(i) $(A \wedge B) \vee (B \wedge C)$
(ii) $A \vee B$
(iii) $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$

Exercise 4.2 (CNF Transformation, Resolution Method)

The following transformation rules hold, whereby propositional formulae can be transformed into equivalent formulae. Here, φ, ψ, and χ are arbitrary propositional formulae:

$$\neg\neg\varphi \equiv \varphi \tag{1}$$
$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi \tag{2}$$
$$\varphi \vee (\psi \wedge \chi) \equiv (\varphi \vee \psi) \wedge (\varphi \vee \chi) \tag{3}$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi \tag{4}$$
$$\varphi \wedge (\psi \vee \chi) \equiv (\varphi \wedge \psi) \vee (\varphi \wedge \chi) \tag{5}$$

Additionally, the operators $\vee$ and $\wedge$ are associative and commutative.

Consider the formula $((C \wedge \neg B) \leftrightarrow A) \wedge (\neg C \rightarrow A)$.

(a) Transform the formula into a clause set K using the CNF transformation rules. Write down the steps.

(b) Afterwards, using the resolution method, show that $K \models (\neg B \rightarrow (A \wedge C))$ holds.

Exercise 4.3 (Davis-Putnam Procedure)

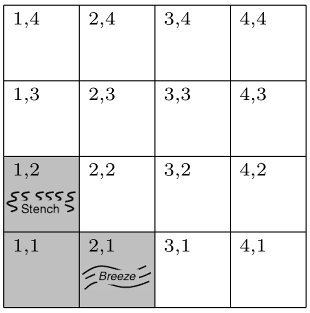
Use the Davis-Putnam procedure to compute models for the following clause sets or to prove that no model exists. Whenever possible, apply the *pure symbol heuristic* (i.e. assignment of the corresponding value to variables always occurring with the same polarity) and *unit propagation*. At each step, indicate which rule you have applied.

(a) $\{\{P, \neg Q\}, \{\neg P, Q\}, \{Q, \neg R\}, \{S\}, \{\neg S, \neg Q, \neg R\}, \{S, R\}\}$

(b) $\{\{P, Q, S, T\}, \{P, S, \neg T\}, \{Q, \neg S, T\}, \{P, \neg S, \neg T\}, \{P, \neg Q\}, \{\neg R, \neg P\}, \{R\}\}$

Exercise 4.4 (Wumpus world and resolution)

Consider the following situation in the wumpus world:



The gray squares have already been visited, the others not. The percepts in the corresponding squares are indicated by Breeze *breeze* and Stench *stench* .

(a) Formalize the general connections between breezes and pits using propositional formulae. Use 16 variables $P_{i,j}$ (meaning there is a *pit* in square $[i, j]$) and 16 variables $B_{i,j}$ (*breeze* in square $[i, j]$).

(b) Show, using *resolution*, that square $[3, 1]$ contains a pit in the given situation, i.e., show that KB $\models P_{3,1}$. The knowledge base KB consists of the propositions from part (a) as well as the percepts of the agent. Note: squares that already have been visited do not contain pits. If necessary, convert the knowledge base into CNF (conjunctive normal form).

The exercise sheets may and should be worked on in groups of three (3) students. Please write all your names and the number of your exercise group on your solution.