Theorem (Kuhn)

Every finite extensive game with perfect information has a SPE.

Proof idea:

- It's constructive. We build an SPE bottom-up (aka backward-induction)
- For those familiar with the AI course: It is a generalization of the minimax procedure.

Pirate game

There are $n \leq 100$ pirates, who have found 100 gold coins that have distributed amongst them. Pirates have strict seniority principle: pirate i is superior to pirate $i+1$.

The most senior pirate proposes a distribution. Then all pirates vote on whether they accept it. In case of a tie, the proposing pirate has casting vote. If they accept, then game ends with distributing the coins. If not, the proposing pirate is thrown over board.

Firstly, pirates prefer to survive. Secondly, they want to maximize coins. Thirdly, all the being the same, they prefer to throw pirates over board.

What happens when 5 rationale pirates play this game?

Pirate 5 is the only surviving one: 100 coins → pirate 5

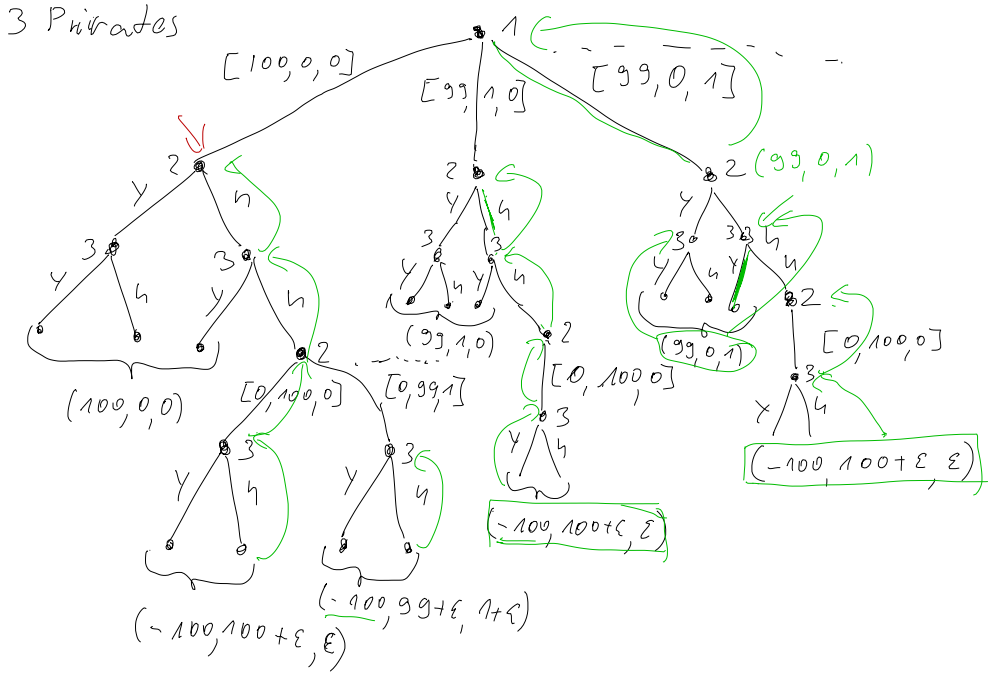
pirate 4, 5 are alive: pirate 4: (100, 0).

pirate 3, 4, 5 are alive: pirate 3: (99, 0, 1).

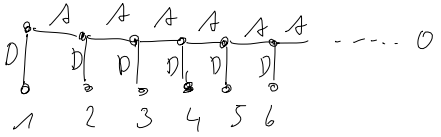
pirate 2, 3, 4, 5 are alive: pirate 2: (99, 0, 1, 0)

pirate 1, 2, 3, 4, 5 are alive: pirate 1: (98, 0, 1, 0, 1).

3 Pirates



Counterexample (infinite horizon):



There is no SPE or NE!

Counterexample (infinite branching)

$A = [0,1)$ with payoffs are $u_1(\langle a \rangle) = a$

Again, no SPE.

Are SPEs unique?

No! However, if no two terminal histories have the same utility, then SPE is unique.

<u>Def</u> An EGWPI and simultaneous moves is a tuple $\Gamma = \langle N, A, H, P, (u_i) \rangle$ where

- $N, A, H,$ and (u_i) are defined as before and
- $P: H \to 2^N$ assigns to each non-terminal history a set of players; for all $h \in H \setminus Z$ there exists a family $(A_i(h))_{i \in P(h)}$ such that

$$A(h) = \{a \mid (h,a) \in H\} = \prod_{i \in P(h)} A_i(h).$$

- What about the one deviation property?
  → still holds
- What about Kuhn's theorem?
  → not necessarily, an SPE mixed strategies in EG would be necessary

<u>Def</u> An EGWPI and chance moves is a tuple $\Gamma = \langle N, A, H, P, f_c, (u_i) \rangle$, where

- N, A, H, u_i are the same as defined before
- the player function $P: H \setminus Z \to N \cup \{c\}$,
- for each $h \in H \setminus Z$ with $P(h) = c$, the function $f_c(\cdot \mid h)$ is a probability measure on $A(h)$ such that the probability measure for all $h \in H$ are independent of each other.

Intended meaning: with chance node, $f_c(\cdot \mid h)$ dictates, what next action to take with a appropriate dice.

- Outcome for a given strategy profile is a probability measure on histories
- Utility is the expected utility.

One deviation property and Kuhn's theorem still hold.