Prisoner's dilemma

Setting: Two prisoners held in separate cells;

prisoners have two options: Confess or don't confess

- if both confess, they will be sentenced to 3 years of prison
- if neither confesses, they will be sentenced to 1 year
- if only one confesses (crown witness) then he does not have to go to jail, the other one gets a sentence of four years.

	Confess	Don't
Confess	-3, -3	0, -4
Don't	-4, 0	-1, -1

$\xrightarrow{+4}$

	C	D
C	1, 1	4, 0
D	0, 4	3, 3

that's because Game theorists don't like negative numbers

Battle of sexes (Bach or Stravinsky)

A coordination game

Setting: Two people want to go together to a concert. They make an appointment for that, but forgot to choose the concert. Player 1 likes Bach and player 2 Stravinsky. Both want to be together.

	B	S
B	2, 1	0, 0
S	0, 0	1, 2

Dove-Hawk or Chicken-game

Setting: Two agent are competing for a resource. They can decide to act like a hawk (attacking) or like a dove (yielding). If both attack, the outcome will be zero, if both yield, they get a moderate payoff, otherwise the hawk gets a high payoff and the dove a little bit.

	H	D
H	0, 0	4, 1
D	1, 4	2, 2

Anti-coordination game

Matching Pennies

Setting: 2 people have to make a choice between "head" and "tail"

If choices differ, then person 1 gives 1 \$ to 2

If choices are the same, then person 2 gives 1 \$ to 1.

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Example of a zero sum game, i.e. the sum of the utilities for all strategy profiles is zero.

Zero sum games are a special case of strictly competitive games: $u_1(a) \leq u_1(b) \iff u_2(a) \geq u_2(b)$

2.4 Solution Concepts & Notation

- what is a "solution" of strategic game?

→ strategy profile, where all players play strategies that are rational.

When talking/writing strategy profiles, we often want talk about a profile, where one player is removed or where original strategy has been replaced.

Notation: Let $a = (a_1, \dots, a_{|N|}) \in \prod_{i \in N} A_i = A$

$A_{-i} := \prod A_{j \in N - \{i\}} = \{(\dots), (\dots), (\dots) \dots\}$

$a_{-i} := (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_{|N|})$

Given strategy profile $a = (a_1, \dots, a_{i-1}, a_i, a_{i+1}, \dots, a_{|N|})$,

let $a_i' \in A_i$, then $(a_{-i}, a_i') = (a_1, \dots, a_{i-1}, a_i', a_{i+1}, \dots, a_{|N|})$.

Example:

$A_1 = \{T, B\}$, $A_2 = \{L, R\}$, $A_3 = \{X, Y, Z\}$

$a = (T, R, Z)$ $\quad a_{-2} = (T, Z)$ $\quad a_{-3} = (T, R)$

$(a_{-2}, L) = (T, L, Z)$

2.5 Elimination of Dominated Strategies

- What strategy should the agent play?

→ Elimate all irrational strategies.

Def A strategy $a_i' \in A_i$ is strictly dominated by strategy $a_i^+ \in A_i$ if for all profiles of the other agents $a_{-i} \in A_{-i}$: $u_i(a_{-i}, a_i') < u_i(a_{-i}, a_i^+)$.

player 1 \ player 2	Conf.	Don't
Conf.	1, 1	4, 0
Don't	0, 4	3, 3

	B	S
B	2, 1	0, 0
S	0, 0	1, 2

Definition

A strategy $a_i' \in A_i$ is <u>weakly dominated</u> by $a_i^+ \in A_i$ if for each $a_{-i} \in A_{-i}$:

$$u_i(a_{-i}, a_i') \leq u_i(a_{-i}, a_i^+)$$

and if for one profile $a_{-i} \in A_{-i}$:

$$u_i(a_{-i}, a_i') < u_i(a_{-i}, a_i^+).$$

	L	R
T	2, 1	0, 0
M	2, 1	1, 1
B	0, 0	1, 1

How to identify an NE?

When you have a payoff matrix, you check wether a deviation makes sense for one agent.

Prisoner's Dilemma

NE

	C	D
C	1, 1	4, 0
D	0, 4	3, 3

NE

	B	S
B	2, 1	0, 0
S	0, 0	1, 2

NE

	H	D
H	0, 0	4, 1
D	1, 4	2, 2

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Alternative Def.

$$B_i(a_{-i}) = \{ a_i \in A_i \mid u_i(a_{-i}, a_i) \geq u_i(a_{-i}, a') \quad \forall a' \in A_i \}$$

$B_i : A_{-i} \to A_i$ is called <u>best response function</u>

A NE a^* is a profile

$$a_i^* \in B(a_{-i}^*);$$

i.e. $a^* \in \prod B_i(a_{-i}^*) =: B(a^*)$ $\qquad a^* \in B(a^*)$

$f(x) = x$ Fixpoint

2.6 Nash Equilibrium

Alternative solution concept: Focus on "stable" profiles – a profile where for each agent it does not make sense to deviate.

Def (Nash Equilibrium)

A Nash equiblivium (NE) of a strategic game G is a strategy profile $a^* \in A$ such that for each $i \in N$:

$$u_i(a^*) \geq u_i(a_{-i}^*, a_i) \quad \text{for all } a_i \in A_i.$$