

Introduction to Game Theory

B. Nebel, R. Mattmüller
T. Schulte
Summer semester 2014

University of Freiburg
Department of Computer Science

Exercise Sheet 10

Due: Thursday, July 24th, 2014

Exercise 10.1 (Theorem of Gibbard-Satterthwaite, 3 Punkte)

Show, without using the theorem of Gibbard-Satterthwaite, that for $|A| \geq 3$ and arbitrarily chosen $a^* \in A$ no incentive compatible social choice function $f : L^n \rightarrow A$ can exist, with

$$f(\prec_1, \dots, \prec_n) = \begin{cases} a, & \text{if } b \prec_i a \text{ for all } b \neq a \text{ and } i = 1, \dots, n \\ a^*, & \text{otherwise.} \end{cases}$$

Exercise 10.2 (Vickrey auction, 2 points)

As introduced in the lecture, the winner of a Vickrey auction¹ has to pay only the price of the second-highest bid. It has been proven that, in case of a single indivisible item, a dominant strategy is for each bidder to bid their true value of this item.

Let's assume that Alice, Bob, and Chris are the sole three competitors in a Vickrey auction about the king's crown. Furthermore, we assume that all three of them possess a nearly infinite amount of ducats, but ascribe different values to the crown, namely:

- For Alice the crown's true value is 52 ducats.
- For Bob the crown's true value is 49 ducats.
- For Chris the crown's true value is 53 ducats.

Explain why for all three competitors both strategies to over- or underbid their respective true values of the crown are dominated by truthful bidding.

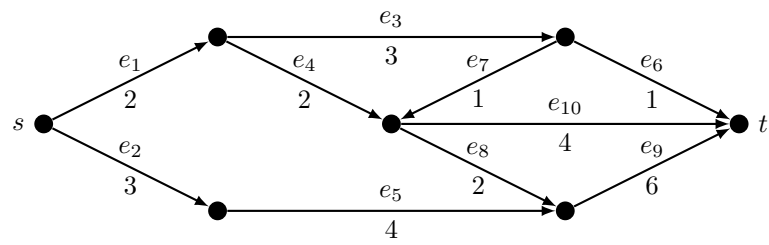
Exercise 10.3 (Network routing as VCG-mechanism, 3 points)

Let $G = (V, E)$ be a directed graph. Every edge $e \in E$ belongs to a player e and generates cost c_e when being used. We want to rent a path between the two nodes s and t . The set of alternatives A contains all paths between s and t . Player e has cost c_e , if edge e lies on the chosen path p , zero otherwise. Maximization of social welfare means minimizing $\sum_{e \in p} c_e$ over all paths p from s to t .

Which alternatives does the VCG-mechanism choose in the following concrete example? Which payments result from this?

Please justify your answers.

¹http://en.wikipedia.org/wiki/Vickrey_auction



The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.