

# Introduction to Game Theory

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## Exercise Sheet 8

**Due: Thursday, July 10th, 2014**

### Exercise 8.1 (Voting procedures, 4 points)

Consider the following voting procedures (for simplicity, we assume that ties are broken in favor of the candidate with the lower index):

**Plurality vote:** Only top preferences are taken into account. The candidate with most top preferences wins.

**Instant runoff voting:** Iteratively candidates with the fewest top preferences are eliminated until only one candidate, the winner, remains.

**Coombs method:** Iteratively candidates with the most bottom (lowest) preferences are eliminated until only one candidate, the winner, remains.

**Borda count:** If a candidate is in position  $j$  of a voter's preference list, he gets  $m - j$  points from that voter. Points from all voters are added. The candidate with most points wins.

Give preference relations  $\prec_1, \dots, \prec_n$  over a candidate set  $A = \{a_1, \dots, a_m\}$  such that the above-mentioned voting procedures return as many different winners as possible. You will obtain one point per different winner.

### Exercise 8.2 (Properties of voting procedures, 4 points)

Consider the voting procedures **plurality vote**, **instant runoff voting**, and the **Borda count**. Again, we assume that ties are broken in favor of the candidate with the lower index. Moreover,  $|A| \geq 3$ . Consider the following properties:

**Majority criterion:** If for more than half of the voters  $i$ ,  $b \prec_i a$  for all  $b \in A \setminus \{a\}$ , then  $f(\prec_1, \dots, \prec_n) = a$ .

**Reversal symmetry:** If  $f(\prec_1, \dots, \prec_n) = a$  and  $a \prec'_i b$  iff  $b \prec_i a$  for all  $i = 1, \dots, n$  and  $a, b \in A$ , then  $f(\prec'_1, \dots, \prec'_n) \neq a$ .

**Incentive compatibility:**  $f(\prec_1, \dots, \prec'_i, \dots, \prec_n) \preceq_i f(\prec_1, \dots, \prec_i, \dots, \prec_n)$  for all  $\prec_1, \dots, \prec_n, \prec'_i \in L$ .

For each of the nine combinations of voting procedure  $f$  and property  $P$ , show that  $f$  satisfies  $P$  or give a counterexample.

The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.