

Introduction to Game Theory

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Exercise Sheet 6

Due: Thursday, June 26th, 2014

Exercise 6.1 (Extensive Games, 2+1+1 points)

The political actors Rosa (R) and Ernesto (E) have to decide whether a political convention should take place in Berlin (b) or Havana (h). Therefore, they consecutively choose one of the two locations. A third person, Karl (K), decides who is allowed to make the first decision (actions r and e). For Rosa and Ernesto the value of each terminal history only depends on the outcome and not on the order in which the decisions were made. Rosa prefers the outcome where she and Ernesto both choose b (utility +2) opposed to the outcome in which both choose h (utility +1) or both choose different locations (utility 0). Ernesto's utilities differ from Rosa only in the way that the roles of b and h are swapped. Karl's preferences are identical to Ernestos.

- (a) Model this situation as an extensive game with perfect information and specify the game tree.
- (b) Specify the set of Rosa's strategies.
- (c) Determine a subgame perfect equilibrium (SPE).

Exercise 6.2 (Decision Order, 2+2 points)

Let $G = \langle \{1, 2\}, (A_i)_{i \in \{1, 2\}}, (u_i)_{i \in \{1, 2\}} \rangle$ be a finite *strategic* game.

We define two extensive two-player games Γ_1 and Γ_2 as follows:

- In Γ_1 player 1 initially selects an action $a_1 \in A_1$, then player 2 selects an action $a_2 \in A_2$. After that the game ends and each player i gets a payoff of $u_i(a_1, a_2)$.
- In Γ_2 player 2 initially selects an action $a_2 \in A_2$, followed by player 1 selecting an action $a_1 \in A_1$. After that the game ends and each player i gets a payoff of $u_i(a_1, a_2)$.

Let s^* be a subgame perfect equilibrium of Γ_1 and r^* be a subgame perfect equilibrium of Γ_2 . Show:

- (a) If G is a ZSG, the following inequality holds: $u_1(O(s^*)) \leq u_1(O(r^*))$.
- (b) In the general case, $u_1(O(s^*)) \leq u_1(O(r^*))$ does not hold.

The exercise sheets may and should be worked on and handed in in groups of two students. Please indicate both names on your solution.