Introduction to Game Theory

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Exercise Sheet 5

Due: Thursday, June 19th, 2014

Exercise 5.1 (Linear Complementary Problem, 1 + 3 points)
Consider the strategic game $G = \langle N, (A_i)_{i \in N}, (u_i)_{i \in N} \rangle$, with

- $N = \{1, 2\}$,
- $A_1 = A_2 = \{r_1, r_2, r_3\}$ and
- utility functions u_1, u_2 as given by the following payoff matrix.

		Player 2		
		r_1	r_2	$r3$
	r_1	$0, 0$	$3, 1$	$3, 3$
Player 1	r_2	$1, 1$	$0, 0$	$1, 3$
	r_3	$1, 1$	$1, 1$	$0, 0$

(a) Determine all pure strategy Nash equilibria for this game.

(b) Determine the mixed strategy Nash equilibria for this game. Proceed as follows:
   1. Formulate the corresponding LCP.
   2. Convert the LCP into a linear program with the following pair of support sets: $(supp(\alpha), supp(\beta)) = (\{r_1, r_2, r_3\}, \{r_1, r_2, r_3\})$.
   3. Solve the linear program and provide values for each $\alpha(r_i)$ and $\beta(r_i)$, $i \in \{1, 2, 3\}$.

(c) What is the expected payoff (u, v) of the NE computed above?

Exercise 5.2 (Naive Algorithm for solving LCPs, 2 + 2 points)
The picnic game consists of two players who have to choose one of five popular picnic places $p_1, \ldots, p_5$. The decision is made independent from each other and the utilities are defined as follows: For $i = j$ the payoffs for both players are 0. For all other strategy profiles (p_i, p_j) the first players payoff is i and the second players payoff is j.

(a) Formalise the picnic game as a strategic game and formulate the corresponding LCP.

(b) Implement the naive algorithm for solving LCPs and determine five different Nash equilibria for the picnic game. Use lp_solve, to solve the linear programs of the resulting sub-problems. Please only provide the five NEs on your exercise sheet and submit your program files to Tim Schulte (`schultet@informatik.uni-freiburg.de`). If you want to use another language than C, C++, Java or Python, contact us first.