Foundations of Artificial Intelligence

6. Board Games

Search Strategies for Games, Games with Chance, State of the Art

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Contents

Why Board Games?

Board games are one of the oldest branches of AI (Shannon and Turing 1950).

- Board games present a very abstract and pure form of competition between two opponents and clearly require a form of "intelligence".
- The states of a game are easy to represent.
- The possible actions of the players are well-defined.

→ Realization of the game as a search problem

→ The individual states are fully accessible

→ It is nonetheless a contingency problem, because the characteristics of the opponent are not known in advance.

Problems

Board games are not only difficult because they are contingency problems, but also because the search trees can become astronomically large.

Examples:

- **Chess**: On average 35 possible actions from every position; often, games have 50 moves per player, resulting in a search depth of 100: $\rightarrow 35^{100} \approx 10^{150}$ nodes in the search tree (with "only" 10^{40} legal chess positions).
- **Go**: On average 200 possible actions with ca. 300 moves $\rightarrow 200^{300} \approx 10^{700}$ nodes.

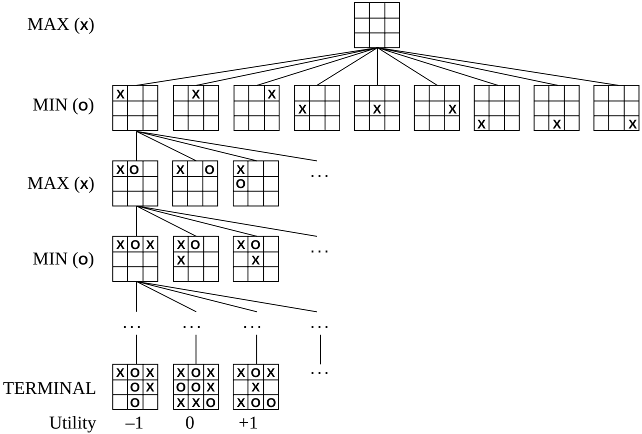
Good game programs have the properties that they

- delete irrelevant branches of the game tree,
- use good evaluation functions for in-between states, and
- look ahead as many moves as possible.

Terminology of Two-Person Board Games

- Players are MAX and MIN, where MAX begins.
- Initial position (e.g., board arrangement)
- Operators (= legal moves)
- Termination test, determines when the game is over. Terminal state = game over.
- Strategy. In contrast to regular searches, where a path from beginning to end is simply a solution, MAX must come up with a strategy to reach a terminal state *regardless of what* MIN *does* → correcting reactions to all of MIN's moves.

Tic-Tac-Toe Example



Every step of the search tree, also called game tree, is given the player's name whose turn it is (MAX- and MIN-steps).

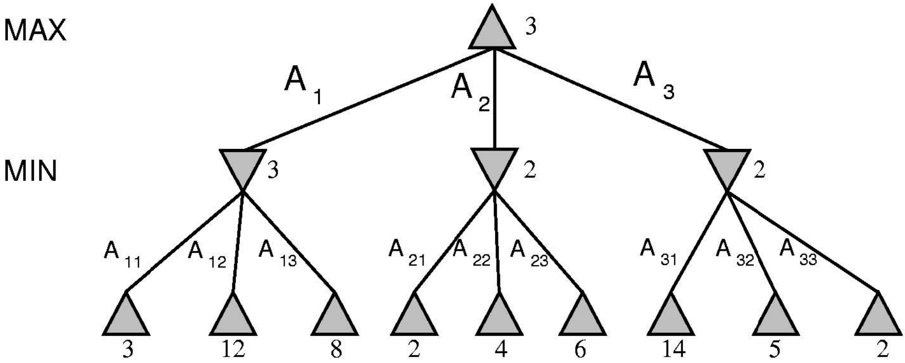
When it is possible, as it is here, to produce the full search tree (game tree), the minimax algorithm delivers an optimal strategy for MAX.

Minimax

1. Generate the complete game tree using depth-first search.
2. Apply the utility function to each terminal state.
3. Beginning with the terminal states, determine the utility of the predecessor nodes as follows:
   - Node is a MIN-node
     Value is the minimum of the successor nodes
   - Node is a MAX-node
     Value is the maximum of the successor nodes
   - From the initial state (root of the game tree), MAX chooses the move that leads to the highest value (minimax decision).

Note: Minimax assumes that MIN plays perfectly. Every weakness (i.e., every mistake MIN makes) can only improve the result for MAX.

Minimax Example



Minimax Algorithm

Recursively calculates the best move from the initial state.

```
function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))
```

```
function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s, a)))
  return v
```

```
function MIN-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
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```

Note: Minimax only works when the game tree is not too deep. Otherwise, the minimax value must be approximated.

Evaluation Function

When the search space is too large, the game tree can be created to a certain depth only. The art is to correctly evaluate the playing position of the leaves.

Example of simple evaluation criteria in chess:

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Example of simple evaluation criteria in chess:

- Material value: pawn 1, knight/bishop 3, rook 5, queen 9
- Other: king safety, good pawn structure
- Rule of thumb: 3-point advantage = certain victory

The choice of the evaluation function is decisive!

The value assigned to a state of play should reflect the chances of winning, i.e., the chance of winning with a 1-point advantage should be less than with a 3-point advantage.

Evaluation Function - General

The preferred evaluation functions are weighted, linear functions:

$$w_1 f_1 + w_2 f_2 + \cdots + w_n f_n$$

where the w's are the weights, and the f's are the features. [e.g., $w_1 = 3$, $f_1 =$ number of our own knights on the board]

The above linear sum makes a strong assumption: the contribution of each feature are independend. (not true: e.g., bishops in the endgame are more powerful, when there is more space)

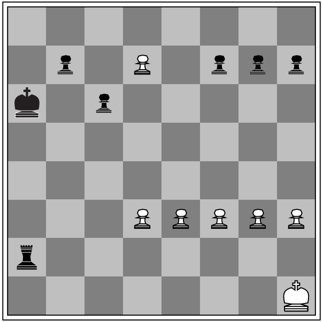
The weights can be learned. The features, however, are often designed by human intuition and understandig

When Should we Stop Growing the Tree?

Motivation: Return an answer within the allocated time.

- Fixed-depth search
- Better: iterative deepening search (stop, when time is over)
- but only stop and evaluate at "quiescent" positions that won't cause large fluctuations in the evaluation function in the following moves. E.g., if one can capture a figure, then the position is not "quiescent" because this might change the evaluation dramatically. Solution: Continue search at non quiescent positions, favorably by only allowing certain types of moves (e.g., capturing) to reduce search effort, until a quiescent position was reached.
- problem of limited depth search: horizon effect (see next slide)

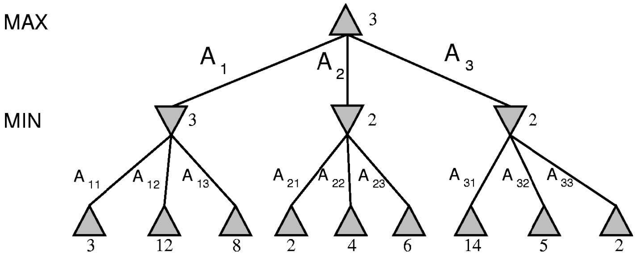
Horizon Problem



Black to move

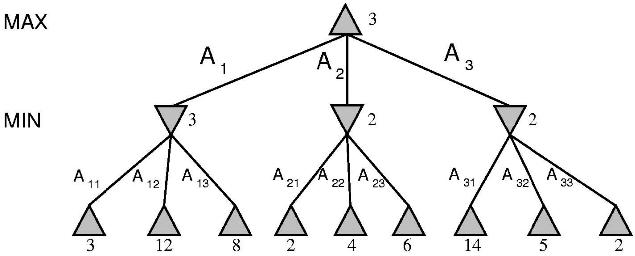
- Black has a slight material advantage
- . . . but will eventually lose (pawn becomes a queen)
- A fixed-depth search cannot detect this because it thinks it can avoid it (on the other side of the horizon - because black is concentrating on the check with the rook, to which white must react).

Alpha-Beta Pruning

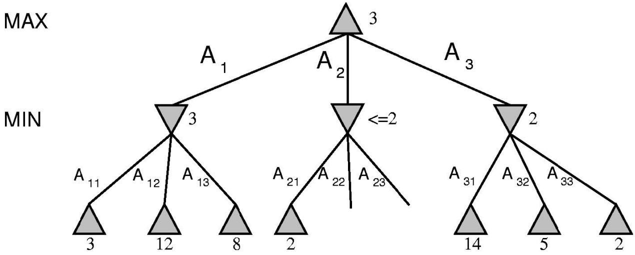


Can we improve this?

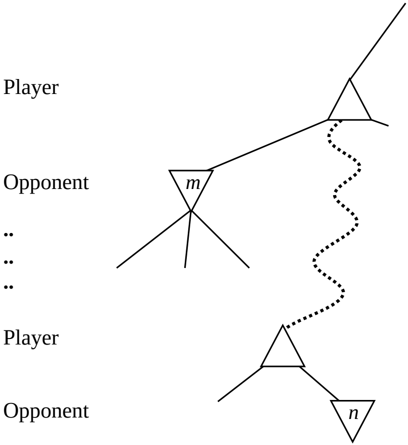
Alpha-Beta Pruning



Can we improve this? We do not need to consider all nodes.



Alpha-Beta Pruning: General



If $m > n$ we will never reach node n in the game.

Alpha-Beta Pruning

Minimax algorithm with depth-first search

$\alpha =$ the value of the best (i.e., highest-value) choice we have found so far at any choice point along the path for MAX.

$\beta =$ the value of the best (i.e., lowest-value) choice we have found so far at any choice point along the path for MIN.

When Can we Prune?

The following applies:

α values of MAX nodes can never decrease

β values of MIN nodes can never increase

(1) Prune below the MIN node whose β-bound is less than or equal to the α-bound of its MAX-predecessor node.

(2) Prune below the MAX node whose α-bound is greater than or equal to the β-bound of its MIN-predecessor node.

$\rightarrow$ Provides the same results as the complete minimax search to the same depth (because only irrelevant nodes are eliminated).

Alpha-Beta Search Algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** an action
  $v \leftarrow$ MAX-VALUE(*state*, $-\infty, +\infty$)
  return the *action* in ACTIONS(*state*) with value v

---

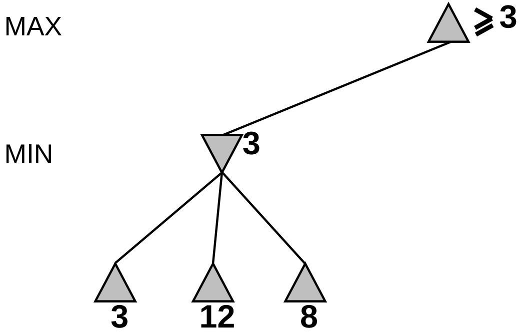
function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

---

function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Initial call with MAX-VALUE(*initial-state*, $-\infty$, $+\infty$)

Alpha-Beta Pruning Example



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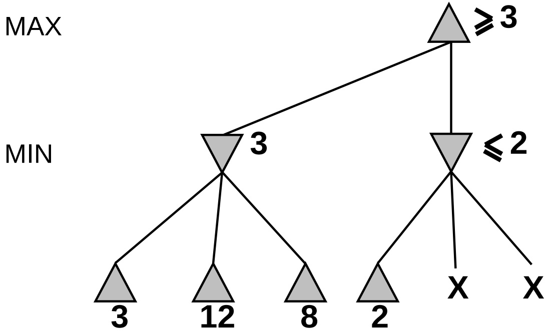

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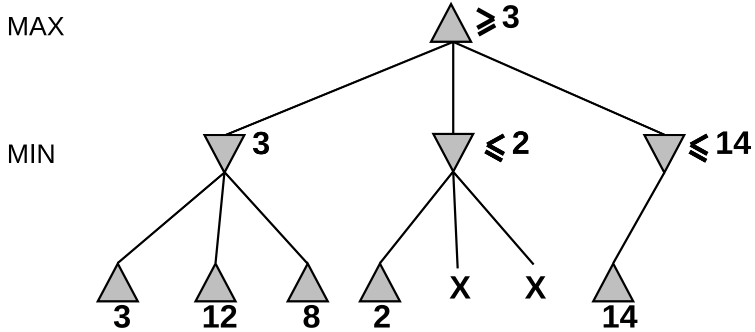

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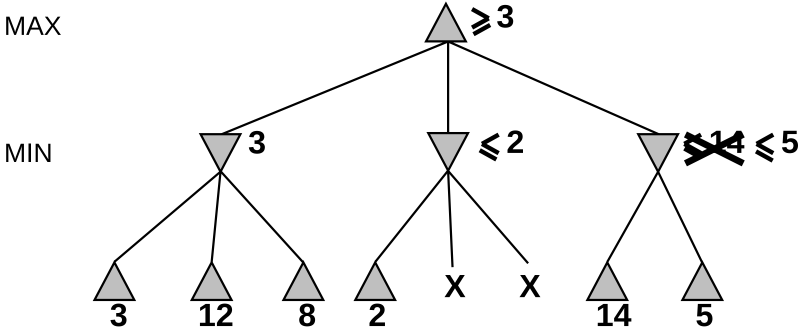
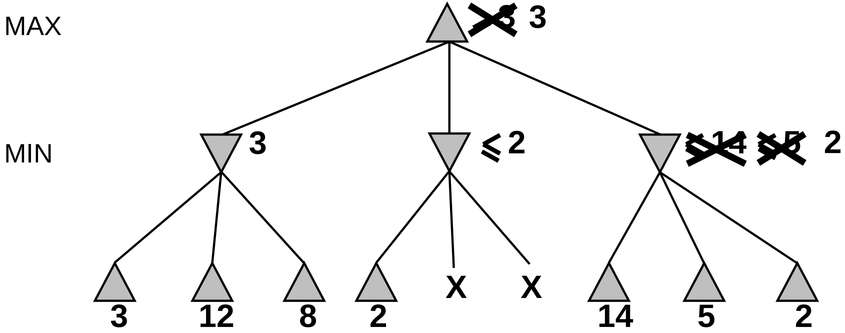


Alpha-Beta Pruning Example



Efficiency Gain

- The alpha-beta search cuts the largest amount off the tree when we examine the best move first.
- In the best case (always the best move first), the search expenditure is reduced to $O(b^{d/2}) \Rightarrow$ we can search twice as deep in the same amount of time.
- In the average case (randomly distributed moves), for moderate b ($b < 100$), we roughly have $O(b^{3d/4})$.
- However, best move typically is not known. Practical case: A simple ordering heuristic brings the performance close to the best case $\Rightarrow$ In chess, we can thus reach a depth of 6-7 moves.

Good ordering for chess?

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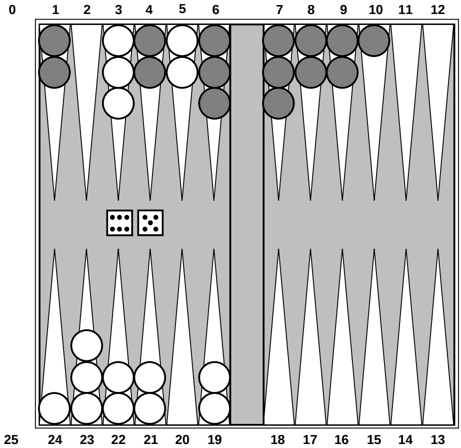
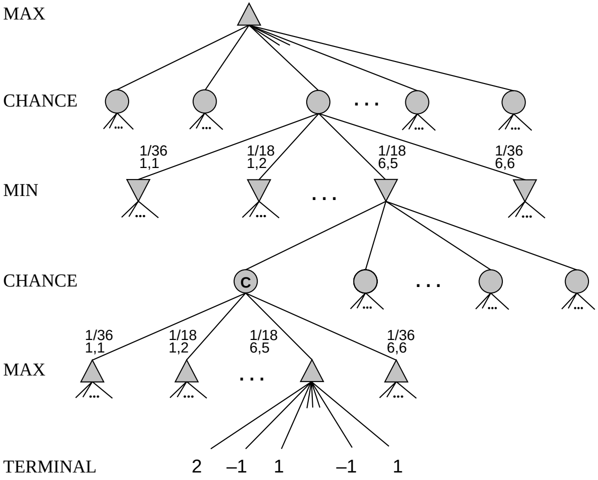
Good ordering for chess? Try captures first, then threats, then forward moves, then backward moves.



White has just rolled 6-5 and has 4 legal moves.

Game Tree for Backgammon

In addition to MIN- and MAX nodes, we need chance nodes (for the dice).



Calculation of the Expected Value

Utility function for chance nodes C over $\textsc{max}$:

d_i: possible dice roll

$P(d_i)$: probability of obtaining that roll

$S(C, d_i)$: attainable positions from C with roll d_i

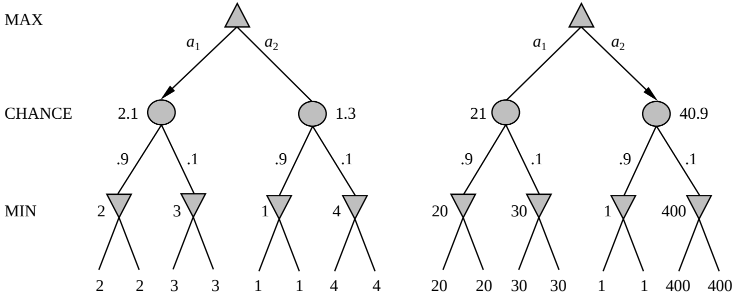
$\textsc{Utility}(s)$: Evaluation of s

$$\textsc{Expectimax}(C) = \sum_i P(d_i) \max_{s \in S(C,d_i)} (\textsc{Utility}(s))$$

$\textsc{Expectimin}$ likewise

Problems

- Order-preserving transformations on the evaluation values may change the best move:



- Search costs increase: Instead of $O(b^d)$, we get $O((b \times n)^d)$, where n is the number of possible dice outcomes.

$\rightarrow$ In Backgammon ($n = 21$, $b = 20$, can be 4000) the maximum for d is 2.

Card Games

- Recently card games such as bridge and poker have been addressed as well
- One approach: simulate play with open cards and then average over all possible plays (or make a Monte Carlo simulation) using minimax (perhaps modified)
- Pick the move with the best expected result (usually all moves will lead to a loss, but some give better results)
  - Averaging over clairvoyancy
- Although "incorrect", appears to give reasonable results

State of the Art

Checkers, **draughts** (by international rules): A program called *CHINOOK* is the official world champion in man-computer competition (acknowledges by ACF and EDA) and the highest-rated player:

CHINOOK: 2712	Ron King: 2632
Asa Long: 2631	Don Lafferty: 2625

Backgammon: The *BKG* program defeated the official world champion in 1980. A newer program TD-Gammon is among the top 3 players.

Othello: Very good, even on normal computers. In 1997, the *Logistello* program defeated the human world champion.

Go: The best programs (Zen, Mogo, Crazystone) using Monte Carlo techniques (UCT) are rated as good as strong amateurs (1kyu/1dan) on the Internet Go servers. However, its usually easy to adapt to the weaknesses of these programs.

Chess (1)

Chess as "Drosophila" of AI research.

- A limited number of rules produces an unlimited number of courses of play. In a game of 40 moves, there are 1.5×10^{128} possible courses of play.
- Victory comes through logic, intuition, creativity, and previous knowledge.
- Only special chess intelligence, no "general knowledge"

Chess (2)

In 1997, world chess master G. Kasparow was beaten by a computer in a match of 6 games.

Deep Blue (IBM Thomas J. Watson Research Center)

- Special hardware (32 processors with 8 chips, 2 Mi. calculations per second)
- Heuristic search
- Case-based reasoning and learning techniques
  - 1996 Knowledge based on 600,000 chess games
  - 1997 Knowledge based on 2 million chess games
  - Training through grand masters
- Duel between the "machine-like human Kasparow vs. the human machine Deep Blue."

Chess (3)

Nowadays, ordinary PC hardware is enough . . .

Name	Strength (ELO)
Rybka 2.3.1	2962
G. Kasperow	2828
V. Anand	2758
A. Karpow	2710
Deep Blue	2680

But note that the machine ELO points are not strictly comparable to human ELO points . . .

The Reasons for Success . . .

- Alpha-Beta-Search
- . . . with dynamic decision-making for uncertain positions
- Good (but usually simple) evaluation functions
- Large databases of opening moves
- Very large game termination databases (for checkers, all 10-piece situations)
- For Go, Monte-Carlo techniques proved to be successful!
- And very fast and parallel processors as well as huge memory!

Summary

- A game can be defined by the initial state, the operators (legal moves), a terminal test and a utility function (outcome of the game).
- In two-player board games, the minimax algorithm can determine the best move by enumerating the entire game tree.
- The alpha-beta algorithm produces the same result but is more efficient because it prunes away irrelevant branches.
- Usually, it is not feasible to construct the complete game tree, so the utility of some states must be determined by an evaluation function.
- Games of chance can be handled by an extension of the alpha-beta algorithm.