Foundations of Artificial Intelligence

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Exercise Sheet 5

Due: Wednesday, June 26, 2013

Exercise 5.1 (Semantics of Predicate Logic)
Consider the Interpretation $\mathcal{I} =< \mathcal{D}, \cdot^{\mathcal{I}} >$ with

- $D = \{0, 1, 2, 3\}$
- $even^{\mathcal{I}} = \{0, 2\}$
- $odd^{\mathcal{I}} = \{1, 3\}$
- $lessThan^{\mathcal{I}} = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$
- $two^{\mathcal{I}} = 2$
- $plus^{\mathcal{I}} : D \times D \to D, plus^{\mathcal{I}}(a, b) = (a + b) \mod 4$

and the variable assignment $\alpha = \{(x, 0), (y, 1)\}$.
Decide for the following formulae θ_i if $\mathcal{I}$ is a model for θ_i under α, i.e. if $\mathcal{I}, \alpha \models \theta_i$. Explain your answer.

(a) $\theta_1 = odd(y) \wedge even(two)$

(b) $\theta_2 = \forall x\ (even(x) \vee odd(x))$

(c) $\theta_3 = \forall x \exists y\ lessThan(x, y)$

(d) $\theta_4 = \forall x\ (even(x) \Rightarrow \exists y\ lessThan(x, y))$

(e) $\theta_5 = \forall x\ (odd(x) \Rightarrow even(plus(x, y)))$

Exercise 5.2 (Normalforms and Herbrand expansion)

(a) Transform the following formula into Skolem Normal Form (SNF):

$$\forall z \exists y (P(x, g(y), z) \vee \neg \forall x Q(x)) \wedge \neg \forall z \exists x \forall t \neg R(f(x, z), z, t)$$

(b) Give the 10 smallest terms in the Herbrand universe and the 5 smallest formulae in the Herbrand expansion of the following formula:

$$\forall x \forall y \forall z\ P(x, f(y, b), g(z))$$

Exercise 5.3 (Substitutions and Unification)

(a) Compute the substitutions

(i) $P(x,y)\{\frac{x}{A}, \frac{y}{f(B)}\}$,

(ii) $P(x,y)\{\frac{x}{f(y)}\}\{\frac{y}{g(B,B)}\}$,

(iii) $P(x,y)\{\frac{x}{f(y)}, \frac{y}{g(B,B)}\}$ and

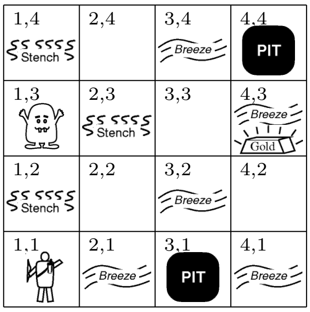
(iv) $P(x,y)\{\frac{z}{f(B)}, \frac{x}{A}\}$

(b) Apply the unification algorithm to the following set of literals:

$$\{R(h(x), f(h(u), y)), R(y, f(y, h(g(A))))\}$$

In each step, give the values of T_k, s_k, D_k, v_k, and t_k.

Exercise 5.4 (Planning in the wumpus world)
Consider the following initial state in the wumpus world:



The agent in square $[1,1]$ did not attend the "Action Planning" lecture, thus, he isn't able to solve planning tasks with partial observability. Additionally he is more excited about hunting the wumpus than about finding gold. Therefore, we define the planning problem as[1]:
Initial state $\mathcal{I}$:

$$\{\texttt{connected}([1,1],[2,1]), \texttt{connected}([2,1],[3,1]), \ldots,$$
$$\texttt{connected}([4,3],[4,4]), \texttt{at}(\texttt{agent},[1,1]), \texttt{at}(\texttt{wumpus},[1,3]),$$
$$\texttt{at}(\texttt{pit},[3,1]), \texttt{at}(\texttt{pit},[4,4]), \texttt{arrowleft}, \texttt{agent\_alive}\}$$

Operators $\mathcal{O}$:

$\texttt{Move}(x,y)$

PRE : $\texttt{at}(\texttt{agent},x) \wedge \texttt{connected}(x,y) \wedge \texttt{agent\_alive}$

EFF : $\texttt{at}(\texttt{wumpus},y) \rhd \neg\texttt{agent\_alive}$,

$\texttt{at}(\texttt{pit},y) \rhd \neg\texttt{agent\_alive}$,

$\texttt{at}(\texttt{agent},y)$,

$\neg\texttt{at}(\texttt{agent},x)$

[1] *stench*,*breeze* and *gold* will not be formalized here and serve only for the purpose of illustration (or confusion?).

$\texttt{Shoot}(x, y)$

$$\begin{aligned} \text{PRE :} & \texttt{at}(\texttt{agent}, x) \land \texttt{connected}(x, y) \land \texttt{arrowleft} \land \texttt{agent\_alive} \\ \text{EFF :} & \texttt{at}(\texttt{wumpus}, y) \rhd \texttt{scream}, \\ & \neg\texttt{arrowleft} \end{aligned}$$

Goal $\mathcal{G}$:

$\texttt{scream} \land \texttt{agent\_alive}$

(a) Suppose, you want to solve a simplified, monotonic planning problem by ignoring negative effects (aka. the "delete relaxation") in order to calculate a heuristic.
Specify the operators of the relaxed planning task.

(b) Sketch the first two levels of the relaxed planning graph. Facts that do not change in the relaxed problem, e.g. $\texttt{agent\_alive}$, $\texttt{at}(\texttt{pit}, x)$ and $\texttt{connected}(x, y)$ can be omitted (In the initial state in layer $F0$ you only have to sketch the fact $\texttt{at}(\texttt{agent}, [1, 1])$).
To further simplify the problem, you may compile away the conditional effect $\texttt{at}(\texttt{wumpus}, y) \rhd \texttt{scream}$ of $\texttt{Shoot}(x, y)$ by moving the effect precondition to the operator precondition[2].

(c) Contrary to the PlanGraph method presented in the lecture, actions cannot be conflicting in a relaxed planning problem since they neither contain negative preconditions nor negative effects. Therefore, relaxed plans can be found more easily and thus be used to derive heuristic estimates.
Specify the relaxed plan. Is this plan also applicable in the original problem?

[2]When compiling away conditional effects, usually two operators (one with the effect condition and one with the negated effect condition) are created. However, $\texttt{Shoot}'(x, y) = \langle \text{PRE} : \texttt{at}(\texttt{agent}, x), \neg\texttt{at}(\texttt{wumpus}, y), \ldots \text{EFF} : \emptyset\rangle$ does not have any effect and might be excluded here as a result.