Introduction to Game Theory

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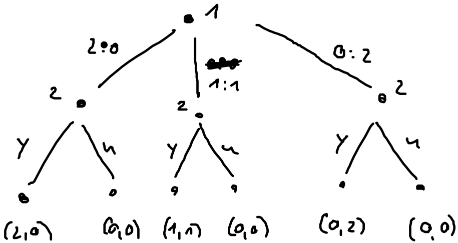
Example (Sharing game)



The only NE without non-credible threats are SPE.

The NE:

$\boxed{(2:0,\ yyy)}$ $\boxed{(1:1,\ nyy)}$

(2:0, yyn) (1:1, nyn)

(2:0, yny) (0:2, nny)

(2:0, ynn)

(2:0, nny)

(2:0, nnn)

3.3 Subgame-perfect Equilibrium

Let $G = \langle N, A, H, \varsigma, (u_i)_{i \in N} \rangle$ be an extensive game with perfect information.

Definition (Subgame):

The subgame of G rooted at history h, $G(h)$, is defined as follows:

$$N' := N$$

$$A' := A$$

$$H' := H|_h := \{ h' : (h, h') \in H \} \qquad Z' := Z|_h$$

$$\varsigma'(h') := \varsigma|_h(h') := \varsigma((h, h')) \qquad h' \in H' \setminus Z'$$

$$u_i'(h') := u_i|_h(h') := u_i((h, h')) \qquad h' \in Z'.$$

For each strategy s_i in G, let $s_i|_h(h') := s_i((h, h'))$ be the induced strategy in $G(h)$. (note this is defined for $h' \in H' \setminus Z'$)

Definition (SPE)

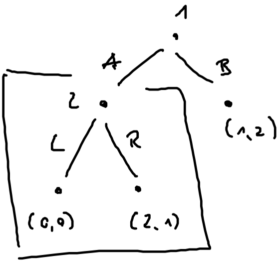
A subgame-perfect equilibrium (SPE) of G is any strategy profile $s = (s_i)_{i\in N}$ such that for each history $h \in H \setminus Z$,

$$s|_h := (s_i|_h)_{i\in N}$$

is a NE of $G(h)$.

Notice: Each SPE is a NE, but of course, the other direction does not hold.

Example:



$s = (A, R)$:

$$s = \left(\{ \emptyset \mapsto A \}, \{ (A) \mapsto R \} \right)$$

$$s|_{(A)} = \left(\{ \emptyset \mapsto A \}|_{(A)}, \{ (A) \mapsto R \}|_{(A)} \right) = \left(- , \{ \emptyset \mapsto R \} \right)$$

$$H = \{ \emptyset, \underline{(A)}, (B), (A,L), (A,R) \}$$

$$H|_{(A)} := \{ \emptyset, (L), (R) \}$$

$\Rightarrow s = (A,R)$: $\left.\begin{cases} h = \emptyset, \text{ in } G(L) \text{ } s \text{ is a NE.} \\ h = (A), \text{ in } G((A)) \text{ } s|_{(A)} \text{ is a NE.} \end{cases}\right\} \Rightarrow$ SPE

$\Rightarrow s = (B,L)$: $\left.\begin{cases} h = \emptyset\text{: in } G(L) \text{ } s \text{ is a NE} \\ h = (A)\text{: in } G((A)) \text{ } s|_{(A)} \text{ is not a NE} \end{cases}\right\} \Rightarrow$ no SPE.