

Introduction to Game Theory

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Sommersemester 2013

Let $G = \langle N, A, H, s, (u_i)_{i \in N} \rangle$ be an extensive game.

Definition (Strategy):

A strategy for player $i \in N$ in G is a function that assigns to each non-terminal history $h \in H \setminus Z$ with $s(h) = i$ an action in $A(h)$.

Definition (Outcome):

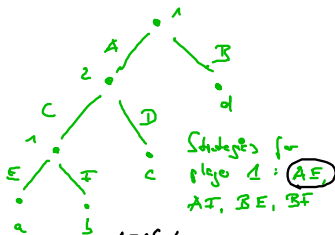
The outcome of a strategy profile $s = (s_i)_{i \in N}$ is the uniquely determined terminal history $h^s = (a^k)_{k=1}^K$ ($K \in \mathbb{N} \cup \{\infty\}$) such that for each $0 \leq k < K$,

$$s_i(a^1, \dots, a^k) = a^{k+1}$$

Let $O(s)$ denote the outcome of profile s .

Notice: Strategy requires to assign actions even when those histories are never played.

Example:



$$AE = \{ \emptyset \mapsto A, (A, C) \mapsto E \}$$

$$AI = \{ (A) \mapsto I \}$$

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3.2 Nash-Equilibria in Extensive Games

Definition (NE)

A Nash equilibrium of an extensive game G with perfect information is a strategy profile $s^* = (s_i^*)_{i \in N}$ such that for each player $i \in N$,

$$s_i^* \in \arg \max_{s_i \in S_i} u_i(\theta(s_i^*, s_{-i}^*)).$$

⇒ Pure strategies

We will see: Each finite extensive game has a pure strategy NE (Zenke, 1913)

Proposition: The NE of G are exactly the NE

of the strategic game induced by G (called its strategic form), which is defined by $G' = (N', (A'_i)_{i \in N}, (u'_i)_{i \in N})$

with:

$$N' = N$$

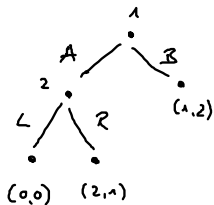
$$A'_i := S_i \quad (i \in N)$$

$$u'_i(s) := u_i(\theta(s))$$

Remarks:

- * Each extensive game with perf information can be transformed into an "equivalent" strategic game, but this can lead to an exponential blowing up of the game representation.
- * The other direction, however, does not hold: there are strategic games (e.g. Prisoner's Dilemma) that have no equivalent representation as a (turn-based) extensive game.
Simultaneous moves cannot always be simulated.

Example :



What are the NE?

	L	R
A	0, 0	2, 1
B	1, 2	1, 2

But the NE (B, L) is not plausible: Choosing B is for player 1 only reasonable if she fears that player 2 plays L.

But given player 1 plays A, player 2 has no reason to play L.

Thus, playing L is a non-credible threat.