Introduction to Game Theory

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2.11 Complexity of Solving Strategic Games

In this section we study the complexity of finding NE to given strategic games that might be subject to further conditions. The basic problem:

NASH : Given a finite 2-player strategic game G, find a mixed strategy profile (α, β) that is a NE of G if one exists, else return "no".

Difference to SAT: Existence is NE is guaranteed.

In this form NASH looks similar to other search problem:

SAT : Given a Boolean formula φ in CNF, find a truth assignment that makes φ true if one exists, else return "no".

A search problem is given by a binary relation $R(x,y)$: Given x, find some y such that $R(x,y)$ holds if such a y exists, else output "no".

Typical complexity classes for search problems:

FP: class of function problems that can be solved by a DTM in polynomial time

FNP: — " — by a NDTM in polynomial time

TFNP: class of function problems in FNP where the function is known to be total.

Proof of existence often based on some "exponential", "unconstructive" method.

Let X be a class of search problems.

A search problem B is called X-complete, if

(a) $B \in X$

(b) Each problem $A \in X$ can be polyno reduced to B,

i.e. there exist functions f mapping instances of A to instances of B and g mapping solutions of B to solutions of A such that:

(i) f and g can be computed in polynomial time

(ii) x is "no" instance of $A \iff f(x)$ is a "no" instance of B

(iii) If y is a solution of B on input $f(x)$, then $g(y)$ is a solution of A on input x.

PPAD: Complexity class (subclass of TFNP) that is specified by the following problem:

Polynomial Parity Argument in Directed Graphs

END-OF-THE-LINE:

Consider a directed graph G with no isolated vertices, each vertex has outdegree and indegree ≤ 1.

Details omitted!

G is specified by two polynomial-time functions f, g that return for each vertex v the predecessor / successor of v, if those exist.

Given a source vertex v in G ($\text{indegree}(v) = 0$), find a vertex $v' \neq v$ that is a source or a sink in G.

$\text{outdegree}(v') = 0$

PPAD then is defined as the class of all search problems in TFNP that can be polyn. reduced to END-OF-THE-LINE.

Theorem (Daskalakis et al., 2006)

NASH is PPAD-complete. ∎

Thus, NASH is presumably "simpler" than the SAT search problem, but presumably "harder" than any polynomial search problem.

2NASH: Given a finite 2-player game G and a NE of G, find a second NE of G, if one exists, else output "no".

Proposition: 2NASH is FNP-complete.

Proof: Reduction from 3SAT.