Introduction to Game Theory

Bernhard Nebel, Robert Mattmüller, Stefan Wölfl, & Christian Becker-Asano

Sommersemester 2013

2.9 Strategic games with incomplete information

So far we considered strategic games in which each player has perfect / complete information about all aspects of the game. In what follows, we introduce games of incomplete information, so called Bayesian games: these games allow us to represent situations in which players are uncertain about "which game is played".

Contrast to games with imperfect information: there all information about the game is known to all the players, but it is not known which moves are made by the other players. Later more on that!

Often it is assumed that there is a "common prior" over all the games the players ~~think~~ conceive as possible, i.e., that there is some mechanism that ensures that all players initially have the same knowledge.

2.9 Strategic games with incomplete information

... Formalize idea that players do not know (exactly) which game is played.

Definition (Bayesian game based on information sets)

A *Bayesian game* is a tuple $\underline{G} = \langle N, G, p, I \rangle$

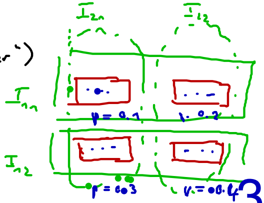
such that

(a) N is a finite set of players;

(b) G is a set of strategic games with player set N such that in all games $g \in G$ all players i have the same set of actions A_i;

(c) p is a probability distribution over G ("common prior")

(d) $I = (I_i)_{i \in N}$ is a family of partitions of G (one for each player i).



A 2nd definition of Bayesian games uses so-called "epistemic types". The epistemic type of a player encapsulated all aspects/information of the game that are available to the player, but are <u>not</u> available to all the other players. When the game is played, each player receives a signal by which the player knows his own type. But the player does not know which signal the other players receive, thus he is uncertain about the type of the other players.

Moreover, we assume that each player has a guess of the types of the other players when he learns about his own type.

<u>Definition</u>: (Bayesian game based on types)

A Bayesian game is a tuple

$G = \langle N, (A_i)_{i\in N}, (\Theta_i)_{i\in N}, (p_i)_{i\in N}, (u_i)_{i\in N} \rangle$

where:

(a) N is a finite set of players;

(b) for each $i \in N$, a finite set of actions A_i;

(c) for each $i \in N$, a finite set of types Θ_i;

(d) for each $i \in N$, $p_i : \Theta_i \to \Delta\, \Theta_{-i}$ assign to each ϑ_i a probability distribution over Θ_{-i}, i.e., $p_i(\vartheta_{-i} \mid \vartheta_i) := p_i(\vartheta_i)(\vartheta_{-i})$ expresses the prob. that all other players have type profile ϑ_{-i}, given that i has type ϑ_i.

(e) for each $i \in N$, $u_i : A \times \Theta \to \mathbb{R}$ utility function.

Remark: Given that the players' initial knowledge is described by a common prior

$$p : \Theta \longrightarrow [0,1]$$

← describes common knowledge! each agent knows of all the other agents the probability of his type

then $p_i : \Theta_i \to \Delta\Theta_{-i}$ is simply defined by

assigns to each θ_i a prob. distribution on Θ_{-i}

$$\theta_i \longmapsto p_i^{\theta_i}$$

with $p_i^{\theta_i} : \Theta_{-i} \to [0,1]$, $p_i^{\theta_i}(\theta_{-i}) := p(\theta_{-i} \mid \theta_i) = \dfrac{p(\theta_{-i}, \theta_i)}{p(\theta_i)}$

← we assume here $p(\theta_i) \neq 0$

Thus, $p_i^{\theta_i}$ expresses agent i beliefs about the other players' types, given he ~~has~~ has learned his type θ_i.

Thus, in some situations it is easier to specify such a common prior than the function $(p_i)_{i\in N}$ mentioned in condition (a)

Example: Version of BoS

Player 2 has complete information about the game, Player 1 does not know whether player 2 likes or dislikes to visit concerts with player 1, but he assumes that it is more likely that player 2 is of type "like" than of type "dislike".

Represent this as follows:

$$N = \{1, 2\} \qquad A_1 = \{B, S\}, \quad A_2 = \{B, S\}$$

$$\Theta_1 = \{t\} \qquad \Theta_2 = \{l, d\}$$

$$p_1(l \mid t) = 0.6, \quad p_1(d \mid t) = 0.4, \quad p_2(t \mid l) = p(t \mid d) = 1$$

	B	S
B	2, 1	0, 0
S	0, 0	1, 2

like ("l")

	B	S
B	2, 0	0, 2
S	0, 1	1, 0

dislike ("d")

Definition: Let G be a Bayesian game

A pure strategy of player i is any mapping

$$s_i : \Theta_i \longrightarrow A_i .$$

A mixed strategy of player is any mapping

$$s_i : \Theta_i \longrightarrow \Delta A_i$$

References

Holler, Manfred J. & Gerhard Illing,
Einführung in die Spieltheorie,
Springer-Verlag, Berlin, 5. Aufl., 2002.

Nisan, Noam, Tim Roughgarden, Éva Tardos, & Vijay V. Vazirani (eds.),
Algorithmic Game Theory,
Cambridge University Press, 2007.

Osborne, Martin J. & Ariel Rubinstein,
A Course in Game Theory,
MIT Press, Cambridge, MA, 1994.