Game theory

B. Nebel, S. Wölfl, R. Mattmüller
C. Becker-Asano, Y. Alkhazraji
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University of Freiburg
Department of Computer Science

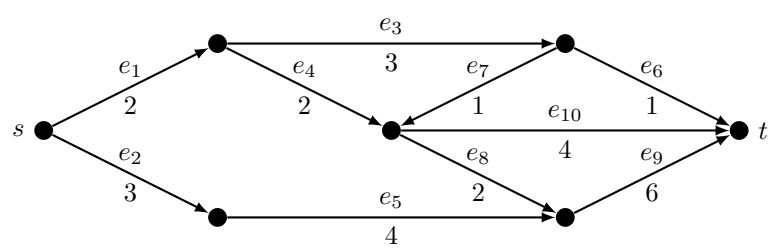
Exercise Sheet 11

Due: Monday, July 15, 2013

Exercise 11.1 (Network routing as VCG-mechanism, 2 points)

Let $G = (V, E)$ be a directed graph. Every edge $e \in E$ belongs to a player e and generates cost c_e when being used. We want to rent a path between the two nodes s and t. The set of alternatives A contains all paths between s and t. Player e has cost c_e, if edge e lies on the chosen path p, zero otherwise. Maximization of social welfare means minimizing $\sum_{e \in p} c_e$ over all paths p from s to t.

Which alternatives does the VCG-mechanism choose in the following concrete example? Which payments result from this?
Please justify your answers.



Exercise 11.2 (VCG-mechanism and incentive compatibility, 3 points)

Let $\mathcal{M} = (f, p_1, \ldots, p_n)$ be a VGC-mechniasm with Clarke-Pivot function. Then $\mathcal{M}$ is incentive compatible and has no positive transfers. Consider the mechanism $\mathcal{M}' = (f, p'_1, \ldots, p'_n)$ mit

$$p'_i(v_1, \ldots, v_n) = p_i(v_1, \ldots, v_n) - \frac{1}{n} \sum_{j=1}^{n} p_j(v_1, \ldots, v_n)$$

for $i = 1, \ldots, n$.

The intuition is that $\mathcal{M}'$ chooses the same alternative as $\mathcal{M}$, the same payments are demanded from the players first, but in the end the excess money will be transferred back to the players in equal shares.

Show by counter example that $\mathcal{M}'$ is not incentive compatible.

Exercise 11.3 (Combinatorial auctions, 3 points)

Apply the greedy-mechanism for single-minded bidders to the following example and state the set of winners, the corresponding allocations, and how much each bidder has to pay.

The bidders are $N = \{1, \ldots, 5\}$ and the items $G = \{1, \ldots, 4\}$. Consider the following bids:

$$
\begin{aligned}
S_1^* &= \{1, 2\} & v_1^* &= 4 \\
S_2^* &= \{3, 4\} & v_2^* &= 4 \\
S_3^* &= \{1\} & v_3^* &= 3 \\
S_4^* &= \{2, 3\} & v_4^* &= 1 \\
S_5^* &= \{4\} & v_5^* &= 3
\end{aligned}
$$

Does the resulting allocation produce optimal social welfare? If not, which allocation would be socially optimal and by which factor do the respective qualities of the calculated allocations differ from that of an optimal solution? Compare the error with the theoretical value of $\sqrt{m}$, with $m = 4$ being the number of goods.