## Game theory

B. Nebel, S. Wölfl, R. Mattmüller
C. Becker-Asano, Y. Alkhazraji
Summer term 2013

University of Freiburg
Department of Computer Science

# Exercise Sheet 9

**Due: Monday, July 1, 2013**

**Exercise 9.1** (Chance moves, 4 points)

Show that substituting one player's moves by chance moves can be beneficial for him or her in a two-player strategic game.
Formally: Define an extensive game $\Gamma = \langle N, H, \rho, (u_i)_{i \in N} \rangle$ and and an extensive game with chance moves $\Gamma' = \langle N, H, \rho', f'_c, (u_i)_{i \in N} \rangle$. The only difference between these two games should be that in $\Gamma'$ the moves of only one player $i \in N$ are substituted by chance moves with uniform probability. Furthermore, player $i$ receives a higher payoff in all subgame perfect equilibria of $\Gamma'$ as compared to those of $\Gamma$.

*Note:* There are examples with only two non-terminal histories.

**Exercise 9.2** (Chain store game with two repetitions, 1+1+2 points)

The *Chain store game with two repetitions* has three players: the chain store owner $L$ and two potential competitors $K_1$ and $K_2$. The game proceeds as follows:

- First, $K_1$ decides whether he opens a shop ($Y$) or not ($N$).
- If $K_1$ opens a shop, $L$ decides, whether to react aggressively ($A$) or passively ($P$).
- After $K_1$ and, if applicable, also $L$ have decided, the same decision process repeats with $K_2$ instead of $K_1$.

If $K_i$ does not open a shop, his payoff is 0. If he opens a shop, his payoff is $-2$, if $L$ reacts aggressively, otherwise it is $+2$.
The payoff for player $L$ is $+6$, if none of the other two players opens a shop. For every shop the other players open his payoff is reduced by 2 and for every aggressive action additionally by 1.

(a) Draw the game tree for the Chain Store Game with two repetitions.

(b) Determine a subgame perfect equilibrium.

(c) Determine a Nash equilibrium, which is not a subgame perfect equilibrium and results in a payoff that is different from that of the equilibrium determined in part (b).