

## Game theory

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### Exercise Sheet 6

**Due: Monday, June 10, 2013**

**Exercise 6.1** (Bayes-Nash equilibria and induced normal form, 2+1+2+1 points)

A Bayes-Nash equilibrium<sup>1</sup> is defined as follows:

A *Bayes-Nash equilibrium* is a mixed-strategy profile  $s$  that satisfies  $\forall i, s_i \in BR_i(s_{-i})$ , with  $BR_i$  denoting the *best response* in a Bayesian game.

Let  $B = (N, G, P, I)$  be a Bayesian Game with  $N = \{1, 2\}$ ,  $G = \{g_1, g_2, g_3, g_4\}$ . The common prior over all four games  $P \in \prod(G)$  and the tuple of partitions  $I = (U, D, L, R)$  over the games  $G$  are given as follows:

		L		R			
		F	B	F	B		
U		F	2,1	2,0	F	2,0	2,1
		B	0,1	2,1	B	0,0	2,1
		$p = 0.25$		$p = 0.25$			
D		F	3,1	2,0	F	3,0	2,1
		B	5,1	4,1	B	5,0	4,1
		$p = 0.25$		$p = 0.25$			

- (a) In order to compute the Bayes-Nash equilibria of  $B$ , first, transform it into its corresponding  $4 \times 4$  normal-form game  $NG$ . (In  $NG$  the payoffs are the expected payoffs in the individual games, given the agents' common prior beliefs. Check Chapter 6.3.3 in the MAS book.)
- (b) Analyze the induced normal form  $NG$  for best responses. For example, what is player one's best response to player two playing strategy RL? How about player two's best responses to player one's strategies?
- (c) Consider the above definition of Bayes-Nash equilibria. Which Bayes-Nash equilibria of  $B$  can you derive from  $NG$  and why?

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<sup>1</sup>See Chapter 6.3 of the Multiagent Systems book by Shoham & Leyton-Brown for further helpful information

- (d) Finally, analyze each single game of  $B$  ( $g_1, g_2, g_3$ , and  $g_4$ ) for their respective Nash-equilibria. Explain their connection to the Bayes-Nash equilibria of  $B$ . Are similar connections guaranteed exist for Bayes-Nash equilibria? Explain.

**Exercise 6.2** (Nash equilibria in zerosum games, 2+2 points)

Let  $G$  be a zerosum game with the following payoff matrix:

	<i>L</i>	<i>C</i>	<i>R</i>
<i>T</i>	-2, 2	3, -3	-4, 4
<i>M</i>	4, -4	-1, 1	1, -1
<i>B</i>	-3, 3	1, -1	-2, 2

- (a) Specify both linear programs that have to be solved to find the strategies for Nash equilibria of this game.
- (b) Use the tool `lp_solve`<sup>2</sup>, to solve the linear programs. Report here the input to `lp_solve` together with its output. Which mixed strategy Nash equilibrium did you find?

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<sup>2</sup><http://sourceforge.net/projects/lpsolve>