Game theory

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Exercise Sheet 4

Due: Thursday, May 16, 2013

Exercise 4.1 (Kakutani's fixed point theorem, 3 points)

Let X be a compact, convex, non-empty subset of $\mathbb{R}^n$ and let $f : X \to \mathcal{P}(X)$ be a set-valued function for which

- for each $x \in X$, the set $f(x)$ is nonempty and convex;
- the graph of f is closed (i.e. for all sequences $\{x_k\}$ and $\{y_k\}$ such that $y_k \in f(x_k)$ for all k, $x_k \to x$, and $y_k \to y$, we have $y \in f(x)$).

Then there exists $x^* \in X$ such that $x^* \in f(x^*)$.

Show that each of the following four conditions is necessary for Kakutani's theorem.

(a) X is compact.

(b) X is convex.

(c) $f(x)$ is convex for each $x \in X$.

(d) f has a closed graph.

Hint: There exists a counter-example with $n = 1$ in each of the four cases.

Exercise 4.2 (Minimax strategy profiles, 1.5+1.5 points)

Let G be a zerosum game with a Nash equilibrium. Use the second part of the theorem that connects the Nash equilibira with minimax strategy profiles of zerosum games to show the following:

(a) If some of the utility values of player 1 are increased such that the resulting game G' is again a zerosum game, then G' does not possess a Nash equilibrium in which player 1 achieves a lower utility than in the Nash equilibrium of G.

(b) If the game G' is constructed from G by deleting an action of player 1, then G' has no Nash equilibrium in which player 1's utility is higher than that achieved in the Nash equlibria of G.

Exercise 4.3 (Nash equilibria in zerosum games, 2 points)

Proof the following claim or give a counter-example: If G is a zerosum game with a Nash equilibrium and player one's payoff is v in this Nash equilibrium, then every strategy profile of G is a Nash quilibrium that results in a payoff of v for player one.

Exercise 4.4 (Mixed strategy Nash equilibrium, 1+1+1 points)

Consider the game "Rock, Stone, Paper, Lizard, Spock" of exercise sheet 1.

(a) Determine the mixed strategy Nash equilibrium for this game, i.e. provide mixed strategies for each player and show that they form a Nash equilibrium of the mixed extension of this game.

(b) Show that this game has no other mixed strategy Nash equilibria.

(c) Verify your results of parts (a) and (b) by using GAMBIT[1] and send the file with the game description to Yusra Alkhazraji (`alkhazry@informatik.uni-freiburg.de`).

ATTENTION:
No lecture on Monday, 13th of May. Next lecture on Thursday, 16th of May.

[1] `http://gambit.sourceforge.net/`