Constraint Satisfaction Problems

B. Nebel, S. Wölfl, J. Hué
M. Westphal
Sommersemester 2012

University of Freiburg
Department of Computer Science

Exercise Sheet 11

Due: Monday 23.07.2012

Exercise 11.1 (3 Points)

Let D be the domain $D = \{0, 1\}$ and Γ the constraint language $\Gamma = \{\neq, R_\vee\}$. Further let d be the ternary Boolean operation defined by

$$d(x, y, z) := (x \wedge y) \vee (y \wedge z) \vee (x \wedge z).$$

Prove Γ is polynomial by the following steps:

(a) Prove the operation d is a polymorphism of Γ.

(b) Prove the operation d is a near-unanimity operation.

Hint: Constraint languages with a near-unanimity operation as polymorphism are polynomial.

Exercise 11.2 (3 Points)

Let $V = (v_1, \dots, v_5)$ be variables with corresponding domains
$D = (\{0\}, \{1, 2\}, \{0, 2, 4\}, \{0, 3\}, \{1, 3, 4\})$.
Establish generalized arc consistency for the *alldifferent*$(v_1, \dots, v_5)$ constraint. For this use the graph-based algorithm presented in the lecture and provide the value graph, the strongly connected components, and the used edges.

Exercise 11.3 (4 Points)

Let $V = (v_1, \dots, v_n)$ be variables with the corresponding domains D_i, $i \in \{1, \dots, n\}$ and G the corresponding value graph.

Prove that the constraint *alldifferent*$(v_1, \dots, v_n)$ is generalized arc-consistent, if and only if every edge in G belongs to a matching in G that covers V.