Constraint Satisfaction Problems

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Exercise Sheet 10

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Exercise 10.1 (1+1+2 Points)

Provide the relations expressed by the following gadgets:

(a) Domain $D = \{1, 2, 3, 4, 5\}$ with the usual ordering of the natural numbers $<$, variables $\{v, w, z\}$, constraints $((v, w), <)$ and $((w, z), <)$, and construction site (z, v).

(b) Domain $D = \{R, G, B, Y\}$, variables $\{a, b, c, d\}$, constraints $((a, b), \neq)$, $((b, c), \neq)$, $((c, d), \neq)$, $((a, c), \neq)$, $((b, d), \neq)$, and construction site (a, d).

(c) Domain $D = \{0, 1, 2, 3, 4\}$, variables $\{x, y, z, v, w\}$, constraints $x + 2y + 3z \equiv 4 \pmod 5$, $2x + z + 3w \equiv 1 \pmod 5$, $z \equiv 3 \pmod 5$, $v + 4y \equiv 2 \pmod 5$ and $x+y+z+v+w \equiv 0 \pmod 5$, and construction site $\{x, z, w\}$.

Exercise 10.2 (6 Points)

Utilize Shaefer's dichotomy theorem to classify each of the following Boolean constraint languages as polynomial or NP-complete. If you classify the language as polynomial, provide which Schaefer class it falls into and an argument for this. For NP-complete cases, you do not have to provide a proof, nor an argument why it does not fall into any of the Schaefer classes.

(a) *Colorability* (with two colors): only inequality constraints

(b) *Parity*: consider k-ary constraints ($k \in \mathbb{N}_1$) that express an even number of variables are assigned 1.

(c) *Balance:* consider k-ary constraints (for even $k \in \mathbb{N}_1$) that express that exactly half of the variables are assigned 1.

(d) *Majority*: consider k-ary constraints (for uneven $k \in \mathbb{N}_1$) that express that the majority of variables are assigned 1.

(e) *One-third*: consider k-ary constraints (for $k \in \mathbb{N}_1$ a multiples of 3) that express exactly one-third of the variables are assigned 1.

(f) *Sheffer stroke*: consider constraints that can be defined based on the binary relation $R_|$ defined by the Boolean function $|$: $\neg x \vee \neg y$.