Constraint Satisfaction Problems

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Exercise Sheet 9

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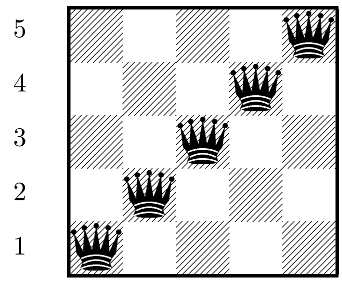
Exercise 9.1 (4 Points)

We consider a variant of stochastic local search (SLS) on the 5-queens problem. The 5-queens problem ask for a placement of 5 queens on a 5×5 chessboard such that no two queens attack each other – this means that no two queens are on the same row, column, or diagonal. For the formalization, assume five variables $V = (q_1, \ldots, q_5)$ representing the queens, which take their row as value. The cost of an assignment is defined as the number of queen pairs that are attacking each other.
For the SLS search consider the following procedure:

(a) Assume the initial state given by $a = (1, 2, 3, 4, 5)$, i.e., the following figure:



$\text{cost}(a) = 10$

(b) Consider changing the row of exactly one queen such that the resulting state is one that has minimal cost among all possible choices (you do not have to prove or argue that it has minimal cost). You can stop the search once a solution or local minimum has been found.

Provide the states (assignments) from the initial state to a solution. For each state also provide its cost.

Exercise 9.2 (3 Points)

Consider the WalkSAT-Procedure with $p = 0.5$ and a satisfiable Boolean constraint network in conjunctive normal form (CNF) as input. Assume that the procedure starts with a random assignment, does not perform restarts, and the number of flips and tries is unlimited.
Prove or disprove that the procedure always finds a solution within *finitely many steps*.

Exercise 9.3 (1+2 Points)

Consider the class of CSP instances CSP($\{T_D\}$), where

$$T_D := \{(0,0,1), (0,1,0), (1,0,0)\}.$$

The 1-in-3 SAT problem is to determine given a propositional logic formula in CNF

$$\varphi = \bigwedge_{i=1}^{n} (l_{i1} \vee l_{i2} \vee l_{i3})$$

whether there is an assignment to variables in which in each clause $(l_{i1} \vee l_{i2} \vee l_{i3})$ exactly one literal is true (and thus the other two are false). Note, in propositional logic a literal is a variable or a negation of a variable.
Prove that CSP($\{T_D\}$) corresponds to 1-in-3 SAT. Show

(a) there is a polynomial-time reduction of CSP($\{T_D\}$) to 1-in-3 SAT,

(b) there is a polynomial-time reduction of 1-in-3 SAT to CSP($\{T_D\}$).

Hint: For (a) show a constraint network with only the relation T_D can be written as a formula φ where only assignments with one true literal per clause are considered. For (b) show φ is expressible as a constraint network with only polynomially many (new) variables and the relation T_D.