Constraint Satisfaction Problems

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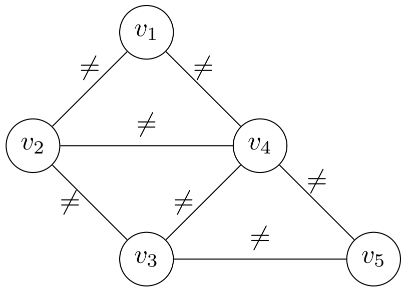
Exercise Sheet 7

Due: Monday 25.06.2012

Exercise 7.1 (2+2 Points)
Consider the constraint network $N = \langle (v_1, \dots, v_5), (D_1, \dots, D_5), C \rangle$ with

- $D_1 = D_4 = \{0, 1, 2\}$,
- $D_2 = \{1, 2\}$,
- $D_3 = \{1, 2, 3\}$,
- $D_5 = \{2, 3\}$.

The constraints C are given by the following constraint graph:

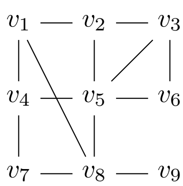


In the following, use the variable ordering $\sigma = v_1, v_2, v_3, v_4, v_5$ for choosing the next unassigned variable. For value selection use the ordering $0, \dots, 3$.
Briefly provide each backtracking step and the reduced domains after constraint propagation at each backtracking step.

(a) Apply the look-ahead algorithm with forward checking to N.

(b) Apply the real full look-ahead algorithm to N, i.e., maintain arc consistency at every search node.

Exercise 7.2 (1+1+1 Points)
Consider the following constraint graph:



For the following questions, every time you pick a variable provide the list of variables that are considered to be best by the algorithm and from this list pick the one with the smallest index.

(a) Use the description from the lecture to find a *max-cardinality* ordering.

(b) Use the description from the lecture to find a *min-width* ordering.

(c) Use the description from the lecture to find a *cycle-cutset* ordering. For this use a cutset of minimal size.

Exercise 7.3 (3 Points)

The algorithm MinWidthOrdering from the lecture provides an ordering of nodes for an undirected graph G with nodes $V = \{v_1, \ldots, v_n\}$. This is done by inductively choosing a node v from V for $j = 1, \ldots, n$ with minimal degree, removing it (and its edges) from the graph and inserting it into the resulting ordering at position $n - j + 1$.
Prove that MinWidthOrdering calculates an ordering which has minimal width.
Hint: A proof can be shown by induction over the size n of the graph G.