Constraint Satisfaction Problems

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Exercise Sheet 6

Due: Monday 18.06.2012

Exercise 6.1 (3 Points)

Let N be a constraint network. Prove that the following three statements are equivalent:

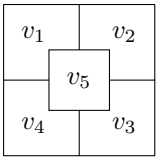
(a) The unordered search space of N is backtrack-free.

(b) The ordered search space of N is backtrack-free along each ordering σ.

(c) N is globally consistent.

Exercise 6.2 (1+2+2+2 Points)

Consider the following coloring problem, where every area is either red, green, or blue and every two areas that share a border must not have the same color.



(a) Formalize this problem as a binary constraint problem N with the regions $V = (v_1, \ldots, v_5)$ as the variables, where every variable has the same domain $D = \{\mathbf{r}, \mathbf{g}, \mathbf{b}\}$.

(b) How many states are in the *unordered search space*? How many goal states (i.e., solutions) are there?

(c) Draw the *ordered search space* for $\sigma_1 = v_1, v_2, v_3, v_4, v_5$. Because of the size (and symmetry) of the tree, you may skip drawing the subtrees starting in $v_1 = \mathbf{b}$ and $v_1 = \mathbf{g}$. Mark the dead ends in your drawing. How many states are in the complete (i.e., including the skipped subtrees) ordered search space? How many are dead ends?

(d) Draw the *ordered search space* for $\sigma_2 = v_5, v_1, v_2, v_3, v_4$. Because of the size (and symmetry) of the tree, you may skip drawing the subtrees starting in $v_5 = \mathbf{b}$ and $v_5 = \mathbf{g}$. How many states are in the complete ordered search space? How many are dead ends?