Constraint Satisfaction Problems

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Exercise Sheet 4

Due: 04.06.2012 **(Note: Submissions now on Mondays)**

Exercise 4.1 (3+1+2 Points)

The zebra problem: There are five houses in a row, each of a different color, inhabited by women of different nationalities. The owner of each house owns a different pet, serves different drinks, and smokes different cigarettes from the other owners. The following facts are also known:

- The Englishwoman lives in the red house.
- The Spaniard owns a dog.
- Coffee is drunk in the green house.
- The Ukrainian drinks tea.
- The green house is immediately to the right of the ivory house.
- The Oldgold smoker owns the snail.
- Kools are smoked in the yellow house.
- Milk is drunk in the middle house.
- The Norwegian lives in the first house on the left.
- The Chesterfield smoker lives next to the fox owner.
- The yellow house is next to the horse owner.
- The Lucky Strike smoker drinks orange juice.
- The Japanese smokes Parliament.
- The Norwegian lives next to the blue house.

The question: Who drinks water and who owns the zebra?
Note: we do not ask for the answer to this question.

(a) Formulate the zebra problem as a binary constraint network $\langle V, D, C \rangle$. Provide the variables V, domains D, and constraints C.

(b) Draw its primal constraint graph.

(c) Is your formalization arc-consistent? If not, provide an equivalent arc-consistent constraint network.

Exercise 4.2 (1+1+2 Points)

Let $\mathcal{C} = \langle V, D, C \rangle$ be a binary constraint network over a finite domain.

(a) Prove that for an input constraint network $\mathcal{C}$ the algorithm AC-3 returns an *equivalent* network $\mathcal{C}'$ (i.e., $\mathcal{C}$ and $\mathcal{C}'$ have the same set of solutions).

(b) Prove that the algorithm AC-3 returns an *arc-consistent* network $\mathcal{C}'$.

(c) Prove that the algorithm AC-4 has a runtime of $\Theta(e \cdot k^2)$.