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## Theory I, Sheet 4

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**Submission: hand in by 8th June 2011 before 16:00**

- The solutions should be submitted in English.
- You must work on your own and write down your own solution. This does not exclude occasional discussions with your fellow students, but solutions copied from other students will not be accepted.

### Exercise 4.1 - AVL trees

[Points: 2]

Consider the AVL tree from exercise 3.1 containing the keys 20, 30, 40, 50, 60, 70. Now:

1. Delete the nodes 40, 60.

Indicate the corresponding operations (rotations, *upin*, *opout* procedure calls).

### Exercise 4.2 - Hashing

[Points: 3]

Take the hash function for strings from the lecture and return the hash code for:

1. "THEORYI", "Hashing" and "Exercise"

Choose an appropriate value for  $m$ . Do you observe any collisions? If no, then find an additional string with a collision, otherwise give a value for  $m$ , such that no collisions can be observed.

### Exercise 4.3 - Universal Hashing

[Points: 5]

Let  $U = \{0, \dots, N - 1\}$ , where  $N$  is 49 and  $m$  is 35. Let  $a_i = 42 \cdot i$  and  $b_i = 28 \cdot i$ . Now consider the following class of hash functions.

$$\mathcal{H} = \{h_i(k) = ((a_i \cdot k + b_i) \bmod N) \bmod m \text{ for } i \in \{1, \dots, N(N - 1)\}\}$$

Is  $\mathcal{H}$  universal? Prove your answer.