Theory I, Sheet 3

Submission: hand in by 1st June 2011 before 16:00

- The solutions should be submitted in English.
- You must work on your own and write down your own solution. This does not exclude occasional discussions with your fellow students, but solutions copied from other students will not be accepted.

Exercise 3.1 - AVL trees [Points: 5]

Consider an empty AVL tree, then:

1. Insert the keys $20, 30, 40, 50, 60, 70$.
2. Delete the nodes $40, 60$.

For every insertion draw the resulting AVL tree as well as the intermediate AVL trees. Indicate the corresponding operations. (rotations, *upin*, *opout* procedure calls).

Exercise 3.2 - AVL trees [Points: 5]

Show by induction the following characterization of Fibonacci numbers:

$$F_h = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{h+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{h+1}\right)$$

Exercise 3.3 - AVL Trees [Points: 2+2+1]

Show that in an AVL tree t with F_k leaves (where F_k is a Fibonacci number) and $k \geq 7$ the *internal path length* $l(t) \leq F_k \cdot (k-4)$. Proceed as follows:

1. Determine the maximum height h of an AVL tree with F_k leaves. Explain informally why in this case an AVL tree with maximum height also has the maxi-mun internal path length.
2. Using induction, show the above inequality.
3. What does this tell us about the average search path length $D(t)$ in such a tree (in terms of its height)?