Theory I, Sheet 1

Submission: hand in by 18th May 2011 before 16:00

- The solutions should be submitted in English.
- You must work on your own and write down your own solution. This does not exclude occasional discussions with your fellow students, but soultions copied from other students will not be accepted.

Exercise 1.2 - Internal Path Length [Points: 5]

The internal path length $l(t)$ of a search tree t is defined as follows:

$$l(t) = \begin{cases} 0 & \text{if } t \text{ is empty} \\ l(t_l) + l(t_r) + size(t) & \text{otherwise} \end{cases}$$

where t_l, t_r are respectively the left and right subtrees of t and $size(t)$ denotes the number of internal nodes of t. Now, let $N(t)$ denote the internal nodes of t. Using induction show that:

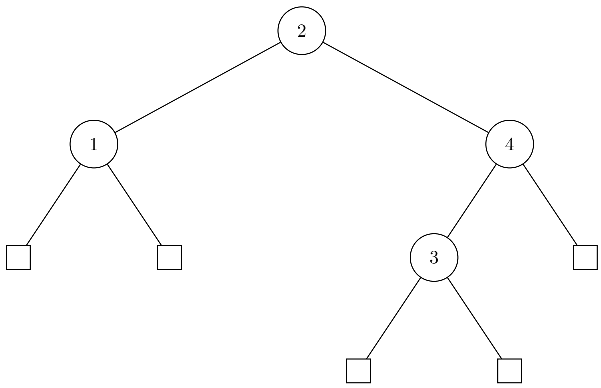
$$l(t) = \sum_{p \in N(t)} (depth(p) + 1)$$

where $depth(p)$ is the distance of node p from the root of t.

Hint: For the base case consider the empty tree. Then, for the inductive step, assume that for a given tree t the desired property holds for subtrees t_l and t_r. Notice that $depth(p)$ has different values depending on the given tree. Assining different names might be helpful (e.g. $depth_t, depth_{t_l}, depth_{t_r}$).

Exercise 1.3 - Trees [Points: 5]

Consider the following Binary Search Tree:



1. Which sequences (permutations) of the keys 1,2,3,4 will result in this shape, if keys are inserted sequentially in an empty tree?
2. Draw all structurally different binary trees with four internal nodes. (Do not draw the leaf nodes).
3. Derive the formula for the number B_N of structurally different binary trees with N internal nodes. Explain your solution.