11 Text search

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Text search

Different scenarios:

Dynamic texts
- Text editors

Static texts
- Literature databases / library systems
- Gene databases
- World Wide Web

Text search

Data type **string**:
- array of character
- file of character
- list of character

Text search

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Operations available: (Let T, P be of type **string**)

Length: length ()
i-**th character**: $T[i]$
concatenation: cat (T, P) $T.P$

Problem definition

Input:

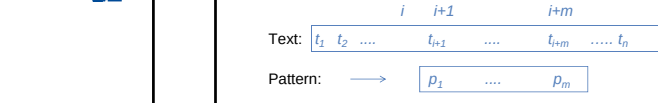
Text $t_1 t_2 \ldots t_n \in \Sigma^n$
Pattern $p_1 p_2 \ldots p_m \in \Sigma^m$

Goal:

Find one or all occurrences of the pattern in the text,
i.e. shifts i $(0 \le i \le n - m)$ such that

$p_1 = t_{i+1}$
$p_2 = t_{i+2}$
$\vdots$
$p_m = t_{i+m}$

Problem definition



Estimation of cost (time) for finding all occurrences:

1. # possible shifts: $n - m + 1$ # pattern positions: m
   → $O(n \cdot m)$

2. At least 1 comparison per m consecutive text positions:
   → $\Omega(m + n/m)$

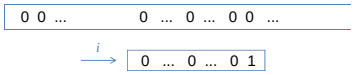
Naïve approach

For each possible shift $0 \le i \le n - m$ check at most m pairs of characters. Whenever a mismatch occurs, start with the next shift.

```
textsearchbf  := proc (T : : string, P : : string)
#  Input:   Text T und pattern P
# Output:  List L of shifts i, at which P occurs in T
    n := length (T); m := length (P);
    L := [];
    for i from 0 to n-m {
          j := 1;
          while j ≤ m and T[i+j] = P[j]
                do j := j+1 od;
          if j = m+1 then L := [L [] , i] fi;
    }
    RETURN (L)
end;
```


Naïve approach

Cost estimation (time):



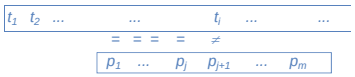
Worst case: $\Omega(m \cdot n)$

In practice: mismatch often occurs very early

→ running time ~ $c \cdot n$

Method of Knuth-Morris-Pratt (KMP)

Let t_i and p_{j+1} be the characters to be compared:



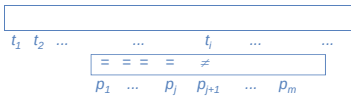
If, at a shift, the first mismatch occurs at t_i and p_{j+1}, then:

- The j characters inspected last in t equal the first j characters in p: $t_{i-j} \ldots t_{i-1} = p_1 \ldots p_j$
- $t_i \neq p_{j+1}$

Method of Knuth-Morris-Pratt (KMP)

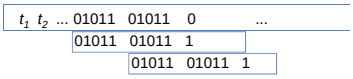
Idea: Compared to the naïve method, we want to improve on two aspects:

- Avoid having to jump back within t. By the fact that $t_{i-j} \ldots t_{i-1} = p_1 \ldots p_j$, all the information we might need about t is also contained in p.
- Determine $j' = next[j] < j$ such that $p_{1 \ldots j'} = t_{i-j'+1 \ldots i}$, and t_i can then be compared with $p_{j'+1}$. I.e., to align t_i with $p_{j'+1}$, shift p by $j-j'$ characters instead of just 1. For this purpose, find the longest prefix of p that is a proper suffix of $p_{1 \ldots j}$, i.e., determine $j' < j$ such that $p_{1 \ldots j'} = p_{j-j'+1 \ldots j}$.



Method of Knuth-Morris-Pratt (KMP)

Example for determining *next*[*j*]:



next[*j*] = length of the longest prefix of P that is a proper suffix of $P_{1 \ldots j}$.

Method of Knuth-Morris-Pratt (KMP)

⇒ for P = 0101101011, *next* = [0,0,1,2,0,1,2,3,4,5] :

1	2	3	4	5	6	7	8	9	10
0	1	0	1	1	0	1	0	1	1
		0							
		0	1						
					0				
					0	1			
					0	1	0		
					0	1	0	1	
					0	1	0	1	1

Method of Knuth-Morris-Pratt (KMP)

```
KMP := proc (T : : string, P : : string)
# Input: text T and pattern P
# Output: list L of shifts i at which P occurs in T
    n := length (T); m := length(P);
    L :=  []; next := KMPnext(P);
    j  :=  0;
    for i from 1 to n do
          while j>0 and T[i] <> P[j+1] do j := next[j] od;
          if T[i]  = P[j+1]  then j := j+1 fi;
          if j = m then L := [L[] , i-m] ;
                        j  := next[j]
          fi;
     od;
     RETURN (L);
end;
```


Method of Knuth-Morris-Pratt (KMP)

Pattern: abracadabra, *next* = [0,0,0,1,0,1,0,1,2,3,4]

```
a b r a c a d a b r a b a b r a c ...
| | | | | | | | | | |
a b r a c a d a b r a
```

next[11] = 4

```
a b r a c a d a b r a b r a b a b r a c ...
              - - - - ↓
              a b r a c
              next[4] = 1
```


Method of Knuth-Morris-Pratt (KMP)

```
a b r a c a d a b r a b  r a b a b r a c ...
                    - | | | ↓
                    a b r a c
                    next [4] = 1

a b r a c a d a b r a b r a b a b r a c ...
                        - | ↓
                        a b r a c
                        next [2] = 0

a b r a c a d a b r a b r a b a b r a c ...
                            | | | | |
                            a b r a c
```


Method of Knuth-Morris-Pratt (KMP)

Correctness:

t_1 t_2 ... ... t_j ... ...
= = = = ≠
p_1 ... p_j p_{j+1} ... p_m

Situation at start of the for-loop:
$p_{1..j} = t_{i-j...i-1}$ and $j \neq m$

if $j = 0$: we are at the first character of p
if $j \neq 0$: p can be shifted (j := next[j]) while $j > 0$ and $t_i \neq p_{j+1}$

Method of Knuth-Morris-Pratt (KMP)

If $t_i = p_{j+1}$, j and i can be increased (at the end of the loop).

When p has been compared completely ($j = m$), a match was found, and we can shift.

Method of Knuth-Morris-Pratt (KMP)

Time complexity:

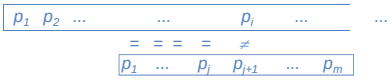
- Text pointer i is never reset
- Text pointer i and pattern pointer j are always incremented together
- Always: *next*[j] < j;
  j can be decreased only as many times as it has been increased.

The KMP algorithm can be carried out in time $O(n)$, if the *next*-array is known.

Computing the *next*-array

next[i] = length of the longest prefix of *P* that is a proper suffix of $P_{1 \ldots i}$.

next[1] = 0
Let *next*[*i*-1] = *j*:



Computing the *next*-array

Consider two cases:

1) $p_i = p_{j+1}$ → *next*[*i*] = *j* + 1

2) $p_i \neq p_{j+1}$ → replace *j* by *next*[*j*] , until $p_i = p_{j+1}$ or *j* = 0.
If $p_i = p_{j+1}$, we can set *next*[*i*] = *j* + 1,
otherwise *next*[*i*] = 0.

Computing the *next*-array

```
KMPnext := proc (P : : string)
#Input    :  pattern P
#Output :   next-Array for P
    m := length (P);
    next := array (1..m);
    next [1] := 0;
    j := 0;
    for i from 2 to m do
       while j > 0 and P[i] <> P[j+1]
           do j := next [j] od;
       if P[i] = P[j+1] then j := j+1 fi;
       next [i] := j
    od;
    RETURN (next);
end;
```


Running time of KMP

The KMP algorithm can be carried out in time O(*n* + *m*).