7 Hashing: chaining

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Possible ways of treating collisions

- Collisions are treated differently in different methods.
- A data set with key *s* is called a colliding element if bucket $B_{h(s)}$ is already taken by another data set.
- What can we do with colliding elements?
  1. Chaining: Implement the buckets as linked lists. Colliding elements are stored in these lists.
  2. Open Addressing: Colliding elements are stored in other vacant buckets. During storage and lookup, these are found through so-called probing.

Chaining (1)

- The hash table is an array (length *m*) of lists.
  Each bucket is implemented by a list.

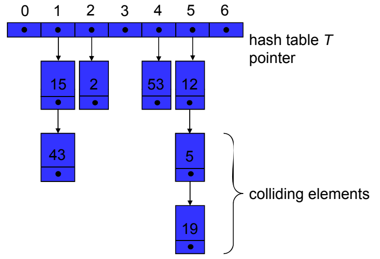
```
class hashTable {
    List[] ht;           // an array of lists
    hashTable (int m){   // Constructor
        ht = new List[m];
        for (int i = 0; i < m; i++)
            ht[i] = new List();  // Construct a list
    }
    ...
}
```

- Two different ways of using lists:
  1. Direct chaining:
  Hash table only contains list headers; all data sets are stored in the lists.
  2. Separate chaining:
  Hash table contains at most one data set in each bucket as well as a list header. Colliding elements are stored in the list.

Direct chaining

Keys are stored in overflow lists

$h(k) = k \bmod 7$



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Chaining

Lookup key *k*
- Compute *h*(*k*) and overflow list *T*[*h*(*k*)]
- Look for *k* in the overflow list

Insert a key *k*
- Lookup *k* (fails)
- Insert *k* in the overflow list

Remove a key *k*
- Lookup *k* (successfully)
- Remove *k* from the overflow list

⟶ only list operations

Implementation in Java

```
class TableEntry {
        private Object key,value;
    }
    abstract class HashTable {
        private TableEntry[] tableEntry;
        private int capacity;
        // Constructor
        HashTable (int capacity) {
            this.capacity = capacity;
            tableEntry = new TableEntry [capacity];
            for (int i = 0; i <= capacity-1; i++)
                tableEntry[i] = null;
        }
        // the hash function
        protected abstract int h (Object key);
        // insert element with given key and value (if not there already)
        public abstract void insert (Object key, Object value);
        // delete element with given key (if there)
        public abstract void delete (Object key);
        // lookup element with given key
        public abstract Object search (Object key);
    } // class hashTable
```

Implementation in Java

```
class ChainedTableEntry extends TableEntry {
        // Constructor
        ChainedTableEntry(Object key, Object value) {
            super(key, value);
            this.next = null;
        }
        private ChainedTableEntry next;
    }
    class ChainedHashTable extends HashTable {
        // the hash function
        public int h(Object key) {
            return key.hashCode() % capacity ;
        }
    // lookup key in the hash table
    public Object search (Object key) {
        ChainedTableEntry p;
        p = (ChainedTableEntry) tableEntry[h(key)];
        // Go through the liste until end reached or key found
        while (p != null && !p.key.equals(key)) {
            p = p.next;
        }
        // Return result
        if (p != null)
            return p.value;
        else return null;
    }
```


Implementation in Java

```
/* Insert an element with given key and value (if not there) */
    public void insert (Object key, Object value) {
        ChainedTableEntry entry = new ChainedTableEntry(key, value);
        // Get table entry for key
        int k = h (key);
        ChainedTableEntry p;
        p = (ChainedTableEntry) tableEntry [k];
        if (p == null){
            tableEntry[k] = entry;
            return ;
        }
        // Lookup key
        while (!p.key.equals(key) && p.next != null) {
            p = p.next;
        }
        // Insert the element (if not there)
        if (!p.key.equals(key))
            p.next = entry;
    }
```


Implementation in Java

```
    // Delete element with given key (if there)
    public void delete (Object key) {
        int k = h (key);
        ChainedTableEntry p;
        p = (ChainedTableEntry) TableEntry [k];
        TableEntry[k] = recDelete(p, key);
    }
    // Delete element with key recursively (if there)
    public ChainedTableEntry recDelete (ChainedTableEntry p, Object key) {
        /* recDelete returns a pointer to the start of the list that p points to,
           in which key was deleted */
        if (p == null)
            return null;
        if (p.key.equals(key))
            return p.getNext();
        // otherwise:
        p.next = recDelete(p.next, key);
        return p;
    }
    public void printTable () {...}
    } // class ChainedHashTable
```


Test program

```
public class ChainedHashingTest {
        public static void main(String args[]){
            Integer[] t= new Integer[args.length];
            for (int i = 0; i < args.length; i++)
                t[i] = Integer.valueOf(args[i]);
            ChainedHashTable h = new ChainedHashTable(7);
            for (int i = 0; i <= t.length - 1; i++)
                h.insert(t[i], null);
            h.printTable ();
            h.delete(t[0]); h.delete(t[1]);
            h.delete(t[6]); h.printTable();
        }
    }
```

Call:

```
java ChainedHashingTest 12 53 5 15 2 19 43
```

Output:

0: -|	0: -|
1: 15 -> 43 -|	1: 15 -|
2: 2 -|	2: 2 -|
3: -|	3: -|
4: 53 -|	4: -|
5: 12 -> 5 -> 19 -|	5: 5 -> 19 -|
6: -|	6: -|

Analysis of direct chaining

Uniform hashing assumption:

- All hash addresses are chosen with the same probability, i.e.:

  $Pr(h(k_i) = j) = 1/m$

- independent from operation to operation

Average chain length for n entries (load factor):

$n/m = \alpha$

Definition:

C'_n = Expected number of entries inspected during a failed search

C_n = Expected number of entries inspected during a successful search

Analysis:

$$C'_n = \alpha$$
$$C_n \approx 1 + \frac{\alpha}{2}$$

Chaining

Advantages:
+ C_n and C'_n are small
+ $\alpha > 1$ possible
+ real distances
+ suitable for secondary memory

Efficiency of lookup

α	C_n (successful)	C'_n (unsuccessful)
0.50	1.250	0.50
0.90	1.450	0.90
0.95	1.457	0.95
1.00	1.500	1.00
2.00	2.000	2.00
3.00	2.500	3.00

Disadvantages:
- Additional space for pointers
- Colliding elements are outside the hash table

Summary

Analysis of hashing with chaining:

- worst case:
  $h(s)$ always yields the same value, all data sets are in a list.
  Behavior as in linear lists.
- average case:
  – Successful lookup & delete:
  complexity (in inspections) ≈ 1 + 0.5 × load factor
  – Failed lookup & insert:
  complexity ≈ load factor
  This holds for direct chaining, with separate chaining the complexity is a bit higher.
- Best case:
  lookup is an immediate success: complexity $\in O(1)$.