Principles of Knowledge Representation and Reasoning

Description Logics – Decidability and Complexity

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Decidability

L_2 is the fragment of first-order predicate logic using only two different variable names (*note*: variable names can be reused!).

$L_2^=$ the same including equality.

Theorem

. $L_2^=$ is decidable.

Corollary

Subsumption and satisfiability of concept descriptions is decidable in description logics using only the following concept and role forming operators: $C \sqcap D$, $C \sqcup D$, $\neg C$, $\forall r.C$, $\exists r.C$, $r \sqsubseteq s$, $r \sqcap s$, $r \sqcup s$, $\neg r$, r^{-1}.

Potential problems: Role composition and cardinality restrictions for role fillers. Cardinality restrictions, however, are not a real problem.

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Undecidability

- $r \circ s$, $r \sqcap s$, $\neg r$, 1 [Schild 88]
- not relevant; Tarski had shown that already! – for relation algebras
- $r \circ s$, $r \doteq s$, $C \sqcap D$, $\forall r.C$ [Schmidt-Schauß 89]
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Decidable, Polynomial-Time Cases

- $\mathcal{FL}^-$ has obviously a polynomial subsumption problem (in the empty TBox) – the SUB algorithm needs only quadratic time.
- Donini *et al* [IJCAI 91] have shown that in the following languages subsumption can be decided using only polynomial time (and they are maximal wrt. this property):

  $C \rightarrow A \,|\, \neg A \,|\, C \sqcap C' \,|\, \forall r.C \,|\, (\geq n\, r) \,|\, (\leq n\, r),\;\; r \rightarrow t \,|\, r^{-1}$
  and
  $C \rightarrow A \,|\, C \sqcap C' \,|\, \forall r.C \,|\, \exists r,\;\; r \rightarrow t \,|\, r^{-1} \,|\, r \sqcap r' \,|\, r \circ r'$
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How Hard is $\mathcal{ALC}$ Subsumption?

Proposition

$\mathcal{ALC}$ subsumption and unsatisfiability are co-NP-hard.

Proof.

Unsatisfiability and subsumption are reducible to each other. We give a reduction from UNSAT. A propositional formula φ over the atoms a_i is mapped to $\pi(\varphi)$:

$$\begin{array}{rcl} a_i & \mapsto & a_i \\ \psi \wedge \psi' & \mapsto & \pi(\psi) \sqcap \pi(\psi') \\ \psi' \vee \psi & \mapsto & \pi(\psi) \sqcup \pi(\psi') \\ \neg\psi & \mapsto & \neg\pi(\psi) \end{array}$$

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How Hard Does It Get?

- Is $\mathcal{ALC}$ unsatisfiability and subsumption also complete for co-NP?
- Unlikely – since models of a single concept description can already become exponentially large!
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Reduction from K-Satisfiability

Lemma (Lower bound for $\mathcal{ALC}$)

$\mathcal{ALC}$ subsumption, unsatisfiability and satisfiability are all PSPACE-hard.

Proof.

Extend the reduction given in the last proof by the following two rules – assuming that b is a fixed role name:

$$\Box\psi \quad \mapsto \quad \forall b.\pi(\psi)$$
$$\Diamond\psi \quad \mapsto \quad \exists b.\pi(\psi)$$

Again, obviously, φ is satisfiable iff $\pi(\varphi)$ is satisfiable (again using structural induction). If φ has a Kripke model, interpret each world w as an object in the universe of discourse that is an instances of the primitive concept $\pi(a_i)$ iff a_i is true in w. For the converse direction use the interpretation the other way around. □

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Computational Complexity of $\mathcal{ALC}$ Subsumption

Lemma (Upper Bound for $\mathcal{ALC}$)

$\mathcal{ALC}$ subsumption, unsatisfiability and satisfiability are all in PSPACE.

Proof.

This follows from the tableau algorithm for $\mathcal{ALC}$. Although there may be exponentially many closed constraint systems, we can visit them step by step generating only one at a time. When closing a system, we have to consider only one role at a time – resulting in an only polynomial space requirement, i.e., satisfiability can be decided in PSPACE. □

Theorem (Complexity of $\mathcal{ALC}$)

$\mathcal{ALC}$ subsumption, unsatisfiability and satisfiability are all PSPACE-complete.

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Further Consequences of the Reducibility of K to $\mathcal{ALC}$

- In the reduction we used only *one* role symbol. Are there modal logics that would require more than one such role symbol?

$\rightsquigarrow$ The **multi-modal logic** $K_{(n)}$ has n different Box operators $\Box_i$ (for n different agents)

$\rightsquigarrow$ $\mathcal{ALC}$ is a *notational variant* of $K_{(n)}$ [Schild, IJCAI-91]

- Are there perhaps other modal logics that correspond to other descriptions logics?

$\rightsquigarrow$ **propositional dynamic logic** (PDL), e.g., transitive closure, composition, role inverse, . . .

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Expressive Power vs. Complexity

- Of course, one wants to have a description logic with high *expressive power*. However, high expressive power implies usually that the **computational complexity** of the reasoning problems might also be high, e.g., $\mathcal{FL}^-$ vs. $\mathcal{ALC}$
- Does it make sense to use a language such as $\mathcal{ALC}$ or even extensions (corresponding to PDL) with higher complexity?
- There are three approaches to this problem:
  1. Use only *small* description logics with *complete* inference algorithms
  2. Use *expressive* description logics, but employ *incomplete* inference algorithms
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Is Subsumption in the Empty TBox Enough?

- We have shown that we can *reduce* concept subsumption in a given TBox to concept subsumption in the empty TBox.
- However, it is not obvious that this can be done in *polynomial time*
- In particular, in the following example *unfolding* leads to an exponential blowup:

$$\begin{array}{rcl} C_1 & \doteq & \forall r.C_0 \sqcap \forall s.C_0 \\ C_2 & \doteq & \forall r.C_1 \sqcap \forall s.C_1 \\ & \vdots & \\ C_n & \doteq & \forall r.C_{n-1} \sqcap \forall s.C_{n-1} \end{array}$$

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TBox Subsumption for Small Languages

- **Question**: Can we decide in polynomial time *TBox subsumption* for a description logic such as $\mathcal{FL}^-$, for which concept subsumption in the empty TBox can be decided in polynomial time?
- Let us consider $\mathcal{FL}_0$: $C \sqcap D, \forall r.C$ with *terminological axioms*.
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Complexity of TBox Subsumption

Theorem (Complexity of TBox subsumption)

TBox subsumption for $\mathcal{FL}_0$ is NP-hard.

Proof sketch.

We use the **NDFA-equivalence problem**, which is NP-complete for *cycle-free* automatons and PSPACE-complete for general NDFAs. We transform a cycle-free NDFA to a $\mathcal{FL}_0$-terminology with the mapping π as follows:

automaton A	$\mapsto$	terminology $\mathcal{T}_A$
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r-transition from q to $q' \mapsto q \doteq \ldots \sqcap \forall r : q' \sqcap \ldots$. □

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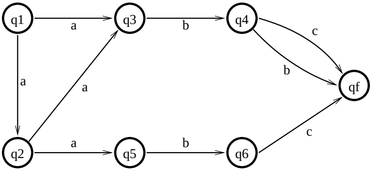
"Proof" by Example



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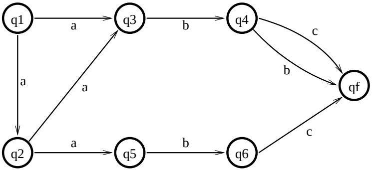

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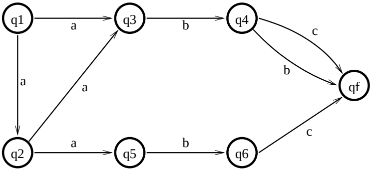

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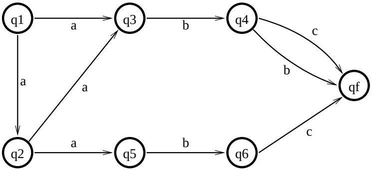

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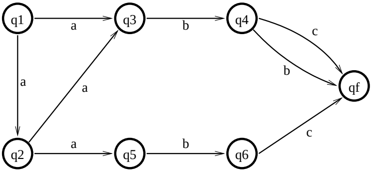

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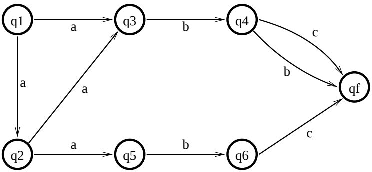

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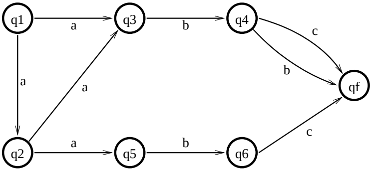

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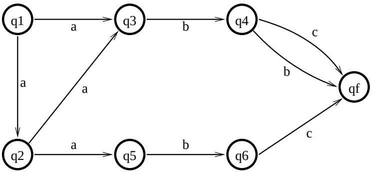

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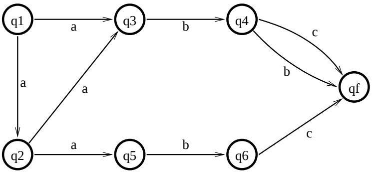

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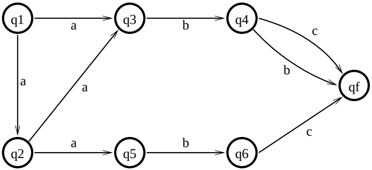

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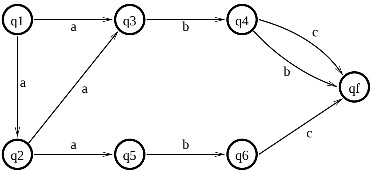

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Outlook

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- Early on, either small languages with provably easy reasoning problems (e.g., the system **CLASSIC**) or large languages with incomplete inference algorithms (e.g., the system **Loom**) were used.
- Meanwhile, one uses complete algorithms on very large descriptions logics (e.g., **SHIQ**), e.g. in the systems **FaCT** and **RACER**
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Literature

- Bernhard Nebel and Gert Smolka. Attributive description formalisms ...and the rest of the world. In Otthein Herzog and Claus-Rainer Rollinger, editors, *Text Understanding in LILOG*, pages 439–452. Springer-Verlag, Berlin, Heidelberg, New York, 1991.
- Francesco M. Donini, Maurizio Lenzerini, Daniele Nardi, and Werner Nutt. Tractable concept languages. In *Proceedings of the 12th International Joint Conference on Artificial Intelligence*, pages 458–465, Sydney, Australia, August 1991. Morgan Kaufmann.
- Klaus Schild. A correspondence theory for terminological logics: Preliminary report. In *Proceedings of the 12th International Joint Conference on Artificial Intelligence*, pages 466–471, Sydney, Australia, August 1991. Morgan Kaufmann.
- I. Horrocks, U. Sattler, and S. Tobies. Reasoning with Individuals for the Description Logic SHIQ. In David MacAllester, ed., *Proceedings of the 17th International Conference on Automated Deduction (CADE-17)*, Germany, 2000. Springer Verlag.
- B. Nebel. Terminological Reasoning is Inherently Intractable, *Artificial*