

Principles of Knowledge Representation and Reasoning

Semantic Networks and Description Logics II: Description Logics – Terminology and Notation

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- Main problem with **semantic networks** and **frames**
- The lack of **formal semantics!**
- Disadvantage of simple **inheritance networks**
- Concepts are atomic and do not have any **structure**

↪ Brachman's **structural inheritance networks** (1977)

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Structural Inheritance Networks

- Concepts are *defined/described* using a small set of well-defined operators
- Distinction between *conceptual* and *object-related* knowledge
- Computation of *subconcept relation* and of *instance relation*
- *Strict inheritance* (of the entire structure of a concept)

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 - **KL-ONE**: First implementation of the ideas (1978)
 - ... then **NIKL, KL-TWO, KRYPTON, KANDOR, CLASSIC, BACK, KRIS, YAK, CRACK** ...
 - ... currently **FaCT, DLP, RACER** 1998
- **Applications:**
 - First, natural language understanding systems
 - ... then configuration systems,
 - ... information systems,
 - ... currently, it is one tool for the *semantic web*
 - **DAML+OIL**, now **OWL**

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Description Logics

- Previously also *KL-ONE-alike languages*, *frame-based languages*, *terminological logics*, *concept languages*
- Description Logics (DL) allow us
 - to describe concepts using *complex descriptions*,
 - to introduce the terminology of an application and to structure it (TBox),
 - to introduce objects (ABox) and relate them to the introduced terminology,
 - and to *reason* about the terminology and the objects.

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Informal Example

Male is: the opposite of female
A **human** is a kind of: living entity
A **woman** is: a human and a female
A **man** is: a human and a male
A **mother** is: a woman with at least one child that is a human
A **father** is: a man with at least one child that is a human
A **parent** is: a mother or a father
A **grandmother** is: a woman, with at least one child that is a parent
A **mother-wod** is: a mother with only male children

Elizabeth is a woman

Elizabeth has the child

Charles

Charles is a man

Diana is a mother-wod

Diana has the child William

Possible Questions:

Is a grandmother a parent?

Is Diana a parent?

Is William a man?

Is Elizabeth a mother-wod?

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Atomic Concepts and Roles

- **Concept names:**
 - E.g., Grandmother, Male, ... (in the following usually *capitalized*)
 - We will use **symbols** such as A, A_1, \dots
 - **Semantics:** Monadic predicates $A(\cdot)$ or set-theoretically a subset of the universe $A^{\mathcal{I}} \subseteq \mathcal{D}$.
- **Role names:**
 - In our example, e.g., child. Often we will use names such as has-child or something similar (in the following usually *lowercase*).
 - Role names are *disjoint* from concept names
 - **Symbolically:** t, t_1, \dots
 - **Semantics:** Dyadic predicates $t(\cdot, \cdot)$ or set-theoretically $t^{\mathcal{I}} \subseteq \mathcal{D} \times \mathcal{D}$.

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Concept and Role Description

- Out of *concept* and *role names*, complex **descriptions** can be created
- In our example, e.g. “a Human and Female.”
- **Symbolically**: C for concept descriptions and r for role descriptions

Which particular constructs are available depends on the chosen description logic!

- **Predicate logic semantics**: A concept descriptions C corresponds to a formula $C(x)$ with the free variable x . Similarly with r : It corresponds to formula $r(x, y)$ with free variables x, y .
- **Set semantics**:

$$C^{\mathcal{I}} = \{d \mid C(d) \text{ “is true in” } \mathcal{I}\}$$

$$r^{\mathcal{I}} = \{(d, e) \mid r(d, e) \text{ “is true in” } \mathcal{I}\}$$

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Boolean Operators

- **Syntax:** let C and D be concept descriptions, then the following are also concept descriptions:
 - $C \sqcap D$ (concept conjunction)
 - $C \sqcup D$ (concept disjunction)
 - $\neg C$ (concept negation)
- **Examples:**
 - Human \sqcap Female
 - Father \sqcup Mother
 - \neg Female
- **Predicate logic semantics:** $C(x) \wedge D(x)$, $C(x) \vee D(x)$, $\neg C(x)$
- **Set semantics:** $C^{\mathcal{I}} \cap D^{\mathcal{I}}$, $C^{\mathcal{I}} \cup D^{\mathcal{I}}$, $\mathcal{D} - C^{\mathcal{I}}$

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 - Human \sqcap Female
 - Father \sqcup Mother
 - \neg Female
- **Predicate logic semantics:** $C(x) \wedge D(x)$, $C(x) \vee D(x)$, $\neg C(x)$
- **Set semantics:** $C^{\mathcal{I}} \cap D^{\mathcal{I}}$, $C^{\mathcal{I}} \cup D^{\mathcal{I}}$, $\mathcal{D} - C^{\mathcal{I}}$

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Boolean Operators

- **Syntax:** let C and D be concept descriptions, then the following are also concept descriptions:
 - $C \sqcap D$ (concept conjunction)
 - $C \sqcup D$ (concept disjunction)
 - $\neg C$ (concept negation)
- **Examples:**
 - Human \sqcap Female
 - Father \sqcup Mother
 - \neg Female
- **Predicate logic semantics:** $C(x) \wedge D(x)$, $C(x) \vee D(x)$, $\neg C(x)$
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Role Restrictions

- **Motivation:**

- Often we want to describe something by *restricting* the possible “fillers” of a role, e.g. Mother-wod.
- Sometimes we want to say that there is at least a filler of a particular type, e.g. Grandmother

- **Idea:** Use **quantifiers** that range over the role-fillers

- $\text{Mother} \sqcap \forall \text{has-child.Man}$
- $\text{Woman} \sqcap \exists \text{has-child.Parent}$

- **Predicate logic semantics:**

$$\begin{aligned}(\exists r.C)(x) &= \exists y : (r(x, y) \wedge C(y)) \\ (\forall r.C)(x) &= \forall y : (r(x, y) \rightarrow C(y))\end{aligned}$$

- **Set semantics:**

$$\begin{aligned}(\exists r.C)^{\mathcal{I}} &= \{d \mid \exists e : (d, e) \in r^{\mathcal{I}} \wedge e \in C^{\mathcal{I}}\} \\ (\forall r.C)^{\mathcal{I}} &= \{d \mid \forall e : (d, e) \in r^{\mathcal{I}} \rightarrow e \in C^{\mathcal{I}}\}\end{aligned}$$

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Cardinality Restriction

- Motivation:

- Often we want to describe something by *restricting the number* of possible “fillers” of a role, e.g., a Mother with at least 3 children or at most 2 children.

- Idea: We restrict the cardinality of the role filler sets:

- Mother $\sqcap (\geq 3 \text{ has-child})$
- Mother $\sqcap (\leq 2 \text{ has-child})$

- Predicate logic semantics:

$$(\geq n r)(x) = \exists y_1 \dots y_n : (r(x, y_1) \wedge \dots \wedge r(x, y_n) \wedge y_1 \neq y_2 \wedge \dots \wedge y_{n-1} \neq y_n)$$

$$(\leq n r)(x) = \neg(\geq n + 1 r)(x)$$

- Set semantics:

$$\begin{aligned}(\geq n r)^{\mathcal{I}} &= \{d \mid |\{e \mid r^{\mathcal{I}}(d, e)\}| \geq n\} \\ (\leq n r)^{\mathcal{I}} &= \mathcal{D} - (\geq n + 1 r)^{\mathcal{I}}\end{aligned}$$

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Inverse Roles

- Motivation:
 - How can we describe the concept “*children of rich parents*”?
- Idea: Define the “inverse” role for a given role (the converse relation)
 - has-child^{-1}
- Application: $\exists \text{has-child}^{-1}.\text{Rich}$
- Predicate logic semantics:

$$r^{-1}(x, y) = r(y, x)$$

- Set semantics:

$$(r^{-1})^{\mathcal{I}} = \{(d, e) \mid (e, d) \in r^{\mathcal{I}}\}$$

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Role Composition

- **Motivation:**
 - How can we define the role `has-grandchild` given the role `has-child`?
- **Idea:** Compose roles (as one can compose binary relations)
 - `has-child` ○ `has-child`
- **Predicate logic semantics:**

$$(r \circ s)(x, y) = \exists z : (r(x, z) \wedge s(z, y))$$

- **Set semantics:**

$$(r \circ s)^{\mathcal{I}} = \{(d, e) \mid \exists f : (d, f) \in r^{\mathcal{I}} \wedge (f, e) \in s^{\mathcal{I}}\}$$

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Role Value Maps

- **Motivation:**
 - How do we express the concept “*women who know all the friends of their children*”
- **Idea:** Relate role filler sets to each other
 - $\text{Woman} \sqcap (\text{has-child} \circ \text{has-friend} \sqsubseteq \text{knows})$

- **Predicate logic semantics:**

$$(r \sqsubseteq s)(x) = \forall y : (r(x, y) \rightarrow s(x, y))$$

- **Set semantics:** Let $r^{\mathcal{I}}(d) = \{e \mid r^{\mathcal{I}}(d, e)\}$.

$$(r \sqsubseteq s)^{\mathcal{I}} = \{d \mid r^{\mathcal{I}}(d) \subseteq s^{\mathcal{I}}(d)\}$$

- **Note:** Role value maps lead to undecidability of satisfiability of concept descriptions!

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Terminology Box

- In order to *introduce* new terms, we use two kinds of **terminological axioms**:

- $A \doteq C$
- $A \sqsubseteq C$

where A is a *concept name* and C is a *concept description*.

- A **terminology** or **TBox** is a finite set of such axioms with the following additional restrictions:
 - no multiple definitions of the same symbol such as $A \doteq C$, $A \sqsubseteq D$
 - no cyclic definitions (even not indirectly), such as $A \doteq \forall r.B$, $B \doteq \exists s.A$

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TBoxes: Semantics

- TBoxes restrict the set of possible interpretations.
- Predicate logic semantics:
 - $A \doteq C$ corresponds to $\forall x : (A(x) \leftrightarrow C(x))$
 - $A \sqsubseteq C$ corresponds to $\forall x : (A(x) \rightarrow C(x))$
- Set semantics:
 - $A \doteq C$ corresponds to $A^{\mathcal{I}} = C^{\mathcal{I}}$
 - $A \sqsubseteq C$ corresponds to $A^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
- Non-empty interpretations which satisfy all terminological axioms are called **models** of the TBox.

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- In order to state something about objects in the world, we use two forms of **assertions**:

- $a : C$
- $(a, b) : r$

where a and b are **individual names** (e.g., ELIZABETH, PHILIP), C is a **concept description**, and r is a **role description**.

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ABoxes: Semantics

- **Individual names** are interpreted as elements of the universe under the **unique-name-assumption**, i.e., different names refer to different objects.
- **Assertions** express that an object is an instance of a concept or that two objects are related by a role.
- **Predicate logic semantics:**
 - $a : C$ corresponds to $C(a)$
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- **Models** of an ABox and of ABox+TBox can be defined analogously to models of a TBox.

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Example TBox

Male $\doteq \neg$ Female
Human \sqsubseteq Living_entity
Woman \doteq Human \sqcap Female
Man \doteq Human \sqcap Male
Mother \doteq Woman $\sqcap \exists$ has-child.Human
Father \doteq Man $\sqcap \exists$ has-child.Human
Parent \doteq Father \sqcup Mother
Grandmother \doteq Woman $\sqcap \exists$ has-child.Parent
Mother-without-daughter \doteq Mother $\sqcap \forall$ has-child.Male
Mother-with-many-children \doteq Mother $\sqcap (\geq 3$ has-child)

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Example ABox

CHARLES: Man

EDWARD: Man

ANDREW: Man

DIANA: Mother-without-daughter

(ELIZABETH, CHARLES): has-child

(ELIZABETH, EDWARD): has-child

(ELIZABETH, ANDREW): has-child

(DIANA, WILLIAM): has-child

(CHARLES, WILLIAM): has-child

DIANA: Woman

ELIZABETH: Woman

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Some Reasoning Services

- Does a description C make sense at all, i.e., is it **satisfiable**?
- A concept description C is satisfiable iff there exists an interpretation \mathcal{I} such that $C^{\mathcal{I}} \neq \emptyset$.
- Is one concept a specialization of another one, is it **subsumed**?
- C is **subsumed by** D , in symbols $C \sqsubseteq D$ iff we have for all interpretations $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.
- Is a an **instance** of a concept C ?
- a is an instance of C iff for all interpretations, we have $a^{\mathcal{I}} \in C^{\mathcal{I}}$.
- **Note:** These questions can be posed with or without a TBox that restricts the possible interpretations.

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- Can we **reduce** the reasoning services to perhaps just one problem?
- What could be **reasoning algorithms**?
- What about **complexity** and **decidability**?
- What has all that to do with **modal logics**?
- How can one build **efficient systems**?

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Summary: Concept Descriptions

Abstract	Concrete	Interpretation
A	A	$A^{\mathcal{I}}$
$C \sqcap D$	(and $C D$)	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
$C \sqcup D$	(or $C D$)	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
$\neg C$	(not C)	$\mathcal{D} - C^{\mathcal{I}}$
$\forall r.C$	(all $r C$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \subseteq C^{\mathcal{I}}\}$
$\exists r$	(some r)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \neq \emptyset\}$
$\geq n r$	(atleast $n r$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \geq n\}$
$\leq n r$	(atmost $n r$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \leq n\}$
$\exists r.C$	(some $r C$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \cap C^{\mathcal{I}} \neq \emptyset\}$
$\geq n r.C$	(atleast $n r C$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \cap C^{\mathcal{I}} \geq n\}$
$\leq n r.C$	(atmost $n r C$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \cap C^{\mathcal{I}} \leq n\}$
$r \doteq s$	(eq $r s$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) = s^{\mathcal{I}}(d)\}$
$r \neq s$	(neq $r s$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \neq s^{\mathcal{I}}(d)\}$
$r \sqsubseteq s$	(subset $r s$)	$\{d \in \mathcal{D} : r^{\mathcal{I}}(d) \subseteq s^{\mathcal{I}}(d)\}$
$g \doteq h$	(eq $g h$)	$\{d \in \mathcal{D} : g^{\mathcal{I}}(d) = h^{\mathcal{I}}(d) \neq \emptyset\}$
$g \neq h$	(neq $g h$)	$\{d \in \mathcal{D} : \emptyset \neq g^{\mathcal{I}}(d) \neq h^{\mathcal{I}}(d) \neq \emptyset\}$
$\{i_1, i_2, \dots, i_n\}$	(oneof $i_1 \dots i_n$)	$\{i_1^{\mathcal{I}}, i_2^{\mathcal{I}}, \dots, i_n^{\mathcal{I}}\}$

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Summary: Role Descriptions

Abstract	Concrete	Interpretation
t	t	$t^{\mathcal{I}}$
f	f	$f^{\mathcal{I}}$, (<i>functional role</i>)
$r \sqcap s$	(and r s)	$r^{\mathcal{I}} \cap s^{\mathcal{I}}$
$r \sqcup s$	(or r s)	$r^{\mathcal{I}} \cup s^{\mathcal{I}}$
$\neg r$	(not r)	$\mathcal{D} \times \mathcal{D} - r^{\mathcal{I}}$
r^{-1}	(inverse r)	$\{(d, d') : (d', d) \in r^{\mathcal{I}}\}$
$r _C$	(restr r C)	$\{(d, d') \in r^{\mathcal{I}} : d' \in C^{\mathcal{I}}\}$
r^+	(trans r)	$(r^{\mathcal{I}})^+$
$r \circ s$	(compose r s)	$r^{\mathcal{I}} \circ s^{\mathcal{I}}$
$\mathbf{1}$	self	$\{(d, d) : d \in \mathcal{D}\}$

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