Principles of Knowledge Representation and Reasoning

Belief Revision

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Belief Change

- A dual approach to nonmonotonic reasoning is belief change.
- We start with some belief state K. When new information arrives, we change the belief state in order to accommodate the new information.
- In the general case, the changed belief state may not be a superset of the original belief state.
- Contrary to nonmonotonic reasoning, here we deal with temporal nonmonotonicity, i.e., the nonmonotonic evolution of a knowledge base or belief state over time.

Two Scenarios

- We have a theory about the world, and the new information is meant to correct our theory ...

$\rightsquigarrow$ belief revision: change your belief state minimally in order to accommodate the new information

- We have a correct theory about the current state of the world, and the new information is meant to record a change in the world ...

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Update and Revision are Different

Assume the new information is consistent with our old beliefs.

- In case of belief revision, we would like to add the new information monotonically to our old beliefs.
- For belief update this is not necessarily the case.
  - Assume we know that the *door is open or the window is open*.
  - Assume we learn that the world has changed and the *door is now closed*.
  - In this case, we do not want to add this information monotonically to our theory, since we would be forced to conclude that *the window is open*.

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Belief Change Operations

General assumption:

- A belief state is modeled by a deductively closed theory, i.e., $K = \mathrm{Cn}(K)$ with Cn the consequence operator
- $\mathcal{L}$: logical language (propositional logic)
- $\mathrm{Th}_{\mathcal{L}}$: set of deductively closed theories (or belief sets) over $\mathcal{L}$

Belief change operations

Monotonic addition: $+ : \mathrm{Th}_{\mathcal{L}} \times \mathcal{L} \to \mathrm{Th}_{\mathcal{L}}$

$K + \psi = \mathrm{Cn}(K \cup \{\psi\})$

Revision: $\dot{+} : \mathrm{Th}_{\mathcal{L}} \times \mathcal{L} \to \mathrm{Th}_{\mathcal{L}}$

Reasonable revision operations?

AGM Revision Postulates (Alchourron, Gärdenfors, Makinson)

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AGM Postulates: Constraining the Space of Revision Operations

AGM postulates:

($\dot{+}$1) $K \dot{+} \varphi \in \mathsf{Th}_{\mathcal{L}}$;

($\dot{+}$2) $\varphi \in K \dot{+} \varphi$;

($\dot{+}$3) $K \dot{+} \varphi \subseteq K + \varphi$;

($\dot{+}$4) If $\neg\varphi \notin K$, then $K + \varphi \subseteq K \dot{+} \varphi$;

($\dot{+}$5) $K \dot{+} \varphi = \mathsf{Cn}(\bot)$ only if $\vdash \neg\varphi$;

($\dot{+}$6) If $\vdash \varphi \leftrightarrow \psi$ then $K \dot{+} \varphi = K \dot{+} \psi$;

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- The postulates constrain the space to fully rational revision operations, but do not pick a single one.
- Revision operations are closed under intersection, so should we choose the minimum?
- NO!

  This is full meet revision, which is known to be useless since $K \dot{+} \varphi = \mathrm{Cn}(\varphi)$ for all φ that are inconsistent with K.
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Belief Revision Schemes

- Preference information (what to keep and what to give up)
- ...may be different for different K's, but independent from the new information φ

$\rightsquigarrow$ compose revision operation pointwise for each K

- In general, a belief revision scheme (BRS) is a "recipe" for deriving a revision operation – restricted to a particular set K – from
  - the belief set and
  - preference information over this belief set

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Examples

Partial meet revision (AGM): Preference information is given by a selection function γ over the set of maximal subtheories consistent with the new information:

$$K \dot{+} \varphi \stackrel{\text{def}}{=} \left(\bigcap \gamma(K \downarrow \neg\varphi)\right) + \varphi,$$

where $K + \varphi = \mathsf{Cn}(K \cup \{\varphi\})$.

Cut revision (GM): Preference information is given by a complete preorder $\preceq$ over all $\psi \in K$:

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Revision – Viewed Computationally

- We don't want to deal with deductively closed theories . . .
- Consider belief bases (finite sets of propositions) to represent belief sets.
- We don't want to specify an arbitrary amount of preference information . . .
- A theory K over the propositional logic $\mathcal{L}$ with n propositional atoms can have as much as
  - 2^{2^n} different propositions,
  - 2^n different models.
- Consider ways of specifying preference information in a concise way, i.e., polynomial in the size of the belief base.

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Base Revision Schemes

- Start with a finite belief base A and preference information over the elements of A ...
- We want to generate a revision operation (restricted to $\mathrm{Cn}(A)$)
- Assume a partitioning of A into n priority classes $A_1, \ldots, A_n$ such that the elements of A_i are more important or relevant than those of A_j for $j < i$
- Equivalently, consider a complete preorder $\trianglelefteq$ over A comparing priorities (epistemic relevance)
- Define a (base) revision scheme that keeps as many of the more relevant propositions as possible

⇒ Base revision schemes generate revision operations in the same way as ordinary schemes do.

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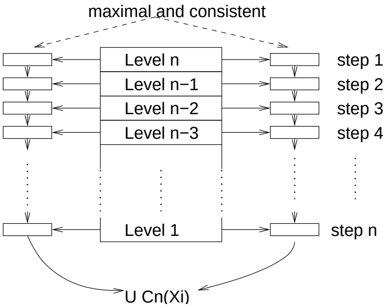

Prioritized Meet-Base Revision

Prioritized Meet-Base Revision (PMBR):

$$A \oplus \varphi \stackrel{\text{def}}{=} \Big(\bigcap_{B \in (A \Downarrow \neg\varphi)} \mathsf{Cn}(B) \Big) + \varphi.$$

Define a revision operation $\dotplus$ on $\mathsf{Cn}(A)$ (that depends on A and the priority information) by

$$\mathsf{Cn}(A) \dotplus \varphi \stackrel{\text{def}}{=} A \oplus \varphi.$$

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Properties of PMBRs

- Generates partial meet revision, but does not satisfy $(\dot{+}8)$ in general.
- Deciding whether $A \oplus \varphi \vdash \psi$ is Π_2^p-complete, even for one priority class.
- A revised base can be represented by

$$A \oplus \varphi = \mathrm{Cn}\Big(\big(\bigvee(A \Downarrow \neg\varphi)\big) \wedge \varphi\Big).$$

- A revised base can become exponentially large:

$$A = \{p_1, \ldots, p_m, q_1, \ldots, q_m\}$$

$$\varphi = \bigwedge_{i=1}^{m} (p_i \leftrightarrow \neg q_i)$$

$(A \Downarrow \varphi)$ has size exponential in $|A|$.

- Worse, in some cases there exists no concise representation of the revised base (provided the polynomial hierarchy does not collapse [Cadoli et al 94]).

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Properties of PMBRs

- Generates partial meet revision, but does not satisfy $(\dot{+}8)$ in general.
- Deciding whether $A \oplus \varphi \vdash \psi$ is Π_2^p-complete, even for one priority class.
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Revision vs. Nonmonotonic Reasoning

Belief Revision and Nonmonotonic Reasoning seem to be of different nature, but there exists a tight connection:

- Given K and a revision operation $\dot{+}$, a nonmonotonic consequence relation can be defined as follows: $\varphi \mid\!\sim \psi$ iff $\psi \in K \dot{+} \varphi$.

In this case,

- the rationality postulates correspond to principles of NMR (such as cautious monotonicity, etc.);
- in the case of prerequisite-free, normal defaults D, the cautions conclusions from (W, D) are simply $D \oplus W$ with one priority level;
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NMR Principles and Rationality Postulates

($\dot{+}$2) $\varphi \in K \dot{+} \varphi$;

- Reflexivity

($\dot{+}$3) $K \dot{+} \varphi \subseteq K + \varphi$;

- Supraclassicality

($\dot{+}$6) If $\vdash \varphi \leftrightarrow \psi$ then $K \dot{+} \varphi = K \dot{+} \psi$;

- Left Logical Equivalence

($\dot{+}$8) If $\neg\psi \notin K \dot{+} \varphi$,
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Conclusions from the Correspondence

- NMR can be thought of as the other side of the same coin.
- NMR (at least for default logic) is as hard as belief revision.
- Representing the conclusions from a propositional default theory using classical propositional logic cannot be done in polynomial space, provided the polynomial hierarchy does not collapse.
- In other words, nonmonotonic logics can be thought of representing (some) information in a denser way than classical logic, and with that come higher computational costs.

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Outlook & Summary

- While NMR and Belief Revision seem to be the two sides of the same coin, there are notable pragmatic differences:
  - Belief revision seems to require that we can easily represent the changed belief base, while for NMR it makes sense to use dense representations.
  - A similar argument could be made for the computational complexity.
- NMR and Belief Revision can be thought of as qualitative ways of dealing with uncertainty in a purely logical setting.
- There exists a strong correspondence between NMR and Belief Revision.
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