

Principles of Knowledge Representation and Reasoning

Belief Revision

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Belief Revision

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Belief Change

- ▶ A dual approach to nonmonotonic reasoning is **belief change**.
- ▶ We start with some **belief state** K . When new information arrives, we change the belief state in order to **accommodate the new information**.
- ▶ In the general case, the changed belief state may not be a superset of the original belief state.
- ▶ Contrary to nonmonotonic reasoning, here we deal with **temporal nonmonotonicity**, i.e., the nonmonotonic evolution of a knowledge base or belief state over time.

Two Scenarios

- ▶ We have a theory about the world, and the new information is meant to **correct** our theory . . .
 - ↔ **belief revision**: change your belief state minimally in order to accommodate the new information
- ▶ We have a correct theory about the current state of the world, and the new information is meant to record a **change** in the world . . .
 - ↔ **belief update**: incorporate the change by assuming that the world has changed minimally

Update and Revision are Different

Assume the new information is consistent with our old beliefs.

- ▶ In case of **belief revision**, we would like to add the new information monotonically to our old beliefs.
- ▶ For **belief update** this is not necessarily the case.
 - Assume we know that the *door is open or the window is open*.
 - Assume we learn that the world has changed and the *door is now closed*.
- ▶ In this case, we do not want to add this information monotonically to our theory, since we would be forced to conclude that *the window is open*.

Belief Change Operations

General assumption:

- ▶ A **belief state** is modeled by a deductively closed theory, i.e., $K = \text{Cn}(K)$ with Cn the **consequence operator**
- ▶ \mathcal{L} : logical language (propositional logic)
- ▶ $\text{Th}_{\mathcal{L}}$: set of deductively closed theories (or belief sets) over \mathcal{L}

Belief change operations

$$\begin{aligned} \text{Monotonic addition: } & + : \text{Th}_{\mathcal{L}} \times \mathcal{L} \rightarrow \text{Th}_{\mathcal{L}} \\ & K + \psi = \text{Cn}(K \cup \{\psi\}) \\ \text{Revision: } & \dot{+} : \text{Th}_{\mathcal{L}} \times \mathcal{L} \rightarrow \text{Th}_{\mathcal{L}} \end{aligned}$$

Reasonable revision operations?

AGM Revision Postulates (Alchourron, Gärdenfors, Makinson)

AGM Postulates: Constraining the Space of Revision Operations

AGM postulates:

- ($\dot{+}$ 1) $K \dot{+} \varphi \in \text{Th}_{\mathcal{L}}$;
- ($\dot{+}$ 2) $\varphi \in K \dot{+} \varphi$;
- ($\dot{+}$ 3) $K \dot{+} \varphi \subseteq K + \varphi$;
- ($\dot{+}$ 4) If $\neg\varphi \notin K$, then $K + \varphi \subseteq K \dot{+} \varphi$;
- ($\dot{+}$ 5) $K \dot{+} \varphi = \text{Cn}(\perp)$ only if $\vdash \neg\varphi$;
- ($\dot{+}$ 6) If $\vdash \varphi \leftrightarrow \psi$ then $K \dot{+} \varphi = K \dot{+} \psi$;
- ($\dot{+}$ 7) $K \dot{+} (\varphi \wedge \psi) \subseteq (K \dot{+} \varphi) + \psi$;
- ($\dot{+}$ 8) If $\neg\psi \notin K \dot{+} \varphi$,
then $(K \dot{+} \varphi) + \psi \subseteq K \dot{+} (\varphi \wedge \psi)$.

Canonical Revision Operations?

- ▶ The postulates **constrain** the space to **fully rational** revision operations, but do not pick a single one.
- ▶ Revision operations are closed under intersection, so should we choose the minimum?
- ▶ **NO!**
This is **full meet revision**, which is known to be useless since $K \dot{+} \varphi = \text{Cn}(\varphi)$ for all φ that are inconsistent with K .
- ▶ What other ways are there to generate a reasonable revision operation?

Belief Revision Schemes

- ▶ Preference information (what to keep and what to give up)
- ▶ ... may be different for different K 's, but independent from the new information φ
- ↪ compose revision operation pointwise for each K
- ▶ In general, a **belief revision scheme** (BRS) is a “recipe” for deriving a revision operation – restricted to a particular set K – from
 - the **belief set** and
 - **preference information** over this belief set

Examples

Partial meet revision (AGM): Preference information is given by a **selection function** γ over the set of **maximal subtheories consistent** with the new information:

$$K \dot{+} \varphi \stackrel{\text{def}}{=} \left(\bigcap \gamma(K \downarrow \neg\varphi) \right) + \varphi,$$

where $K + \varphi = \text{Cn}(K \cup \{\varphi\})$.

Cut revision (GM): Preference information is given by a complete preorder \preceq over all $\psi \in K$:

$$K \dot{+} \varphi \stackrel{\text{def}}{=} \{ \psi \in K \mid \neg\varphi \prec \psi \} + \varphi.$$

Provided \preceq satisfies a number of axioms (**epistemic entrenchment**), cut revisions correspond to **fully rational** revision operations.

Revision – Viewed Computationally

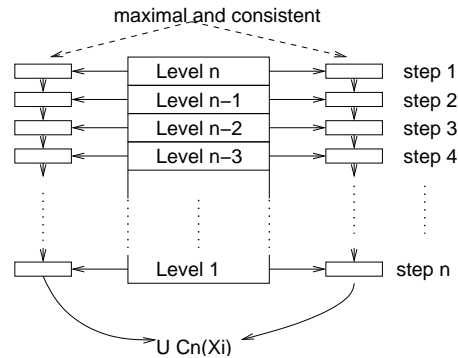
- ▶ We don't want to deal with deductively closed theories ...
- ▶ Consider **belief bases** (finite sets of propositions) to **represent** belief sets.
- ▶ We don't want to specify an arbitrary amount of preference information ...
- ▶ A theory K over the propositional logic \mathcal{L} with n propositional atoms can have as much as
 - ▶ 2^{2^n} different propositions,
 - ▶ 2^n different models.
- ▶ Consider ways of specifying preference information in a **concise** way, i.e., polynomial in the size of the belief base.

Base Revision Schemes

- ▶ Start with a **finite belief base** A and **preference information** over the elements of A ...
- ▶ We want to generate a revision operation (restricted to $\text{Cn}(A)$)
- ▶ Assume a partitioning of A into n **priority classes** A_1, \dots, A_n such that the elements of A_j are more important or relevant than those of A_i for $j < i$
- ▶ Equivalently, consider a complete preorder \preceq over A comparing priorities (**epistemic relevance**)
- ▶ Define a (**base**) **revision scheme** that keeps as many of the more relevant propositions as possible
- ⇒ Base revision schemes generate revision operations in the same way as ordinary schemes do.

Example: Prioritized Meet-Base Revision

Let $(A \Downarrow \neg\varphi)$ be the maximal subsets of A that are consistent with φ and maximize relevant propositions.



Prioritized Meet-Base Revision

Prioritized Meet-Base Revision (PMBR):

$$A \oplus \varphi \stackrel{\text{def}}{=} \left(\bigcap_{B \in (A \Downarrow \neg\varphi)} \text{Cn}(B) \right) + \varphi.$$

Define a **revision operation** $\dot{+}$ on $\text{Cn}(A)$ (that depends on A and the priority information) by

$$\text{Cn}(A) \dot{+} \varphi \stackrel{\text{def}}{=} A \oplus \varphi.$$

Properties of PMBRs

- ▶ Generates **partial meet revision**, but does not satisfy $(\dot{+}8)$ in general.
- ▶ Deciding whether $A \oplus \varphi \vdash \psi$ is Π_2^P -complete, even for one priority class.
- ▶ A **revised base** can be represented by

$$A \oplus \varphi = \text{Cn}\left(\left(\bigvee(A \Downarrow \neg\varphi)\right) \wedge \varphi\right).$$

- ▶ A revised base can become **exponentially large**:

$$A = \{p_1, \dots, p_m, q_1, \dots, q_m\}$$

$$\varphi = \bigwedge_{i=1}^m (p_i \leftrightarrow \neg q_i)$$

$(A \Downarrow \varphi)$ has size exponential in $|A|$.

- ▶ Worse, in some cases there exists no concise representation of the revised base (provided the polynomial hierarchy does not collapse [Cadoli et al 94]).

Revision vs. Nonmonotonic Reasoning

Belief Revision and Nonmonotonic Reasoning seem to be of different nature, but there exists a tight connection:

- ▶ Given K and a revision operation $\dot{+}$, a **nonmonotonic consequence relation** can be defined as follows:
 $\varphi \vdash \psi$ iff $\psi \in K \dot{+} \varphi$.

In this case,

- ▶ the **rationality postulates** correspond to **principles** of NMR (such as cautious monotonicity, etc.);
- ▶ in the case of prerequisite-free, normal defaults D , the cautious conclusions from (W, D) are simply $D \oplus W$ with one priority level;
- ▶ a similar relationship holds between **Brewka's level default theories** and **PMBRs**.

NMR Principles and Rationality Postulates

- (+2) $\varphi \in K \dot{+} \varphi$;
 - ▶ Reflexivity
- (+3) $K \dot{+} \varphi \subseteq K + \varphi$;
 - ▶ Supraclassicality
- (+6) If $\vdash \varphi \leftrightarrow \psi$ then $K \dot{+} \varphi = K \dot{+} \psi$;
 - ▶ Left Logical Equivalence
- (+8) If $\neg\psi \notin K \dot{+} \varphi$,
then $(K \dot{+} \varphi) + \psi \subseteq K \dot{+} (\varphi \wedge \psi)$;
 - ▶ Rational Monotonicity





Conclusions from the Correspondence

- ▶ NMR can be thought of as the other side of the same coin.
- ▶ NMR (at least for default logic) is **as hard as** belief revision.
- ▶ Representing the conclusions from a propositional default theory using classical propositional logic cannot be done in **polynomial space**, provided the polynomial hierarchy does not collapse.
- ▶ In other words, nonmonotonic logics can be thought of representing (some) information in a **denser** way than classical logic, and with that come higher computational costs.

Outlook & Summary

- ▶ While NMR and Belief Revision seem to be the two sides of the same coin, there are notable **pragmatic differences**:
 - ▶ Belief revision seems to require that we can easily represent the changed belief base, while for NMR it makes sense to use **dense representations**.
 - ▶ A similar argument could be made for the **computational complexity**.
- ▶ NMR and Belief Revision can be thought of as **qualitative ways** of dealing with uncertainty in a purely logical setting.
- ▶ There exists a strong **correspondence** between **NMR** and **Belief Revision**.
- ▶ Both are computationally expensive and representational problematic.
- ▶ There are cases, though, that are tractable and practical.

Literature I

-  **D. Makinson.**
How to give it up: A survey of some formal aspects of theory change.
Synthese, 62:347–363, 1985.
Very good introduction to the topic.
-  **B. Nebel.**
Belief revision and default reasoning: Syntax-based approaches.
In *KR-91*, pages 417–428, Cambridge, MA, Apr. 1991.
-  **C. E. Alchourrón, P. Gärdenfors, and D. Makinson.**
On the logic of theory change: Partial meet contraction and revision functions.
Journal of Symbolic Logic, 50(2):510–530, June 1985.
Introduces the so-called AGM approaches: Characterizing belief revision operations by postulates.
-  **P. Gärdenfors.**
Knowledge in Flux—Modeling the Dynamics of Epistemic States.
MIT Press, Cambridge, MA, 1988.

Literature II



B. Nebel.

How hard is it to revise a belief base?

In D. Dubois and H. Prade (eds.), *Handbook of Defeasible Reasoning and Uncertainty Management Systems, Vol. 3: Belief Change*, Kluwer Academic, Dordrecht, The Netherlands, 1998, 77-145.



P. Gärdenfors.

Belief revision and nonmonotonic logic: Two sides of the same coin?

ECAI-90, 768-773.



H. Rott.

Change, choice and inference: A study of belief revision and nonmonotonic reasoning,

Clarendon, Oxford, 2001.