

# Principles of Knowledge Representation and Reasoning

## Nonmonotonic Reasoning

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# A Motivating Example: Defaults in Knowledge Bases

- 1 `employee(anne)`
- 2 `employee(bert)`
- 3 `employee(carla)`
- 4 `employee(detlef)`
- 5 `employee(thomas)`
- 6 `onUnpaidMPaternityLeave(thomas)`
- 7 `employee(X)  $\wedge$   $\neg$  onUnpaidMPaternityLeave(X)  $\rightarrow$  gettingSalary(X)`
- 8 typically: `employee(X)  $\rightarrow$   $\neg$  onUnpaidMPaternityLeave(X)`

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# A Motivating Example: Common Sense Reasoning

- 1 Tweety is a bird like other birds.
- 2 During the summer he stays in Northern Europe, in the winter he stays in Africa.
  - Would you expect Tweety to be able to fly?
  - How does Tweety get from Northern Europe to Africa?

How would you formalize this in formal logic so that you get the expected answers?

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# A Formalization . . .

- 1  $\text{bird}(\text{tweety})$
  - 2  $\text{spend-summer}(\text{tweety}, \text{northern-europe}) \wedge \text{spend-winter}(\text{tweety}, \text{africa})$
  - 3  $\forall x (\text{bird}(x) \rightarrow \text{can-fly}(x))$
  - 4  $\text{far-away}(\text{northern-europe}, \text{africa})$
  - 5  $\forall xyz (\text{can-fly}(x) \wedge \text{far-away}(y, z) \wedge \text{spend-summer}(x, y) \wedge \text{spend-winter}(x, z) \rightarrow \text{flies}(x, y, z))$
- The implication (3) is just a **reasonable assumption**
  - What if Tweety is an **emu**?

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# Examples of Such Reasoning Patterns

**Closed world assumption:** Data-base of **ground atoms**. All ground atoms not present are **assumed** to be false.

**Negation as failure:** In PROLOG, **NOT(P)** means “P is not **provable**” instead of “P is provably false”.

**Non-strict inheritance:** An attribute value is **inherited** only if there is no more specialized information contradicting the attribute value.

**Reasoning about actions:** When reasoning about actions, it is usually assumed that a property **changes** only if it **has to change**, i.e., properties by default do not change.

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# Default, Defeasible, and Nonmonotonic Reasoning

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**Default Reasoning:** Jump to a conclusion if there is no information that contradicts the conclusion.

**Defeasible Reasoning:** Reasoning based on assumptions that can turn out to be wrong, — i.e., conclusions are defeasible. In particular, default reasoning is defeasible.

**Nonmonotonic Reasoning:** In classical logic, the set of consequences grows monotonically with the set of premises. If reasoning is defeasible, then reasoning becomes nonmonotonic.

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# Approaches to Nonmonotonic Reasoning

- **Consistency-based:** Extend classical theory by rules that test whether an assumption is consistent with existing beliefs
- ⇒ nonmonotonic logics like **DL** (default logic), **NMLP** (nonmonotonic logic programming)
- **Entailment-based on normal models:** Models are ordered by **normality**. Entailment is determined by considering the most normal models only.
- ⇒ **Circumscription, Preferential and Cumulative Logics**

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# NM Logic – Consistency-Based

If  $\varphi$  typically implies  $\psi$ ,  $\varphi$  is given, and it is consistent to assume  $\psi$ , then conclude  $\psi$ .

① Typically  $\text{bird}(x)$  implies  $\text{can-fly}(x)$

②  $\forall x(\text{emu}(x) \rightarrow \text{bird}(x))$

③  $\forall x(\text{emu}(x) \rightarrow \neg \text{can-fly}(x))$

④  $\text{bird}(\text{tweety})$

$\Rightarrow \text{can-fly}(\text{tweety})$

⑤ ... +  $\text{emu}(\text{tweety})$

$\Rightarrow \neg \text{can-fly}(\text{tweety})$

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# NM Logic – Normal Models

If  $\varphi$  typically implies  $\psi$ , then the models satisfying  $\varphi \wedge \psi$  should be more normal than those satisfying  $\varphi \wedge \neg\psi$ .

Similarly, try to minimize the interpretation of “Abnormality” predicates.

- 1  $\forall x(\text{bird}(x) \wedge \neg\text{Ab}(x) \rightarrow \text{can-fly}(x))$
- 2  $\forall x(\text{emu}(x) \rightarrow \text{bird}(x))$
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- 4  $\text{bird}(\text{tweety})$

Minimize interpretation of Ab.

$\Rightarrow \text{can-fly}(\text{tweety})$

- 5 ... +  $\text{emu}(\text{tweety})$

$\Rightarrow$  Now in all models (incl. the normal ones):  $\neg \text{can-fly}(\text{tweety})$

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# Default Logic – Outline

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  - Basics
  - Extensions
  - Properties of Extensions
  - Normal Defaults
  - Default Proofs
  - Decidability
  - Propositional DL
- 3 Complexity of Default Logic
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# Motivation: Reiter's Default Logic

- We want to express something like “typically birds fly”.
- Add **non-logical inference rule**

$$\frac{\text{bird}(x) : \text{can-fly}(x)}{\text{can-fly}(x)}$$

with the **intended meaning**:

*If  $x$  is a bird and if it is consistent to assume that  $x$  can fly, then conclude that  $x$  can fly.*

- **Exceptions** can be represented as formulae:

$$\begin{aligned}\forall x(\text{penguin}(x) &\rightarrow \neg\text{can-fly}(x)) \\ \forall x(\text{emu}(x) &\rightarrow \neg\text{can-fly}(x)) \\ \forall x(\text{kiwi}(x) &\rightarrow \neg\text{can-fly}(x))\end{aligned}$$

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# Formal Framework

- **FOL** with classical provability relation  $\vdash$  and deductive closure:  $\text{Th}(\Phi) := \{\phi \mid \Phi \vdash \phi\}$
- **Default rules:**  $\frac{\alpha: \beta}{\gamma}$ 
  - $\alpha$ : **Prerequisite:** must have been derived before rule can be applied.
  - $\beta$ : **Consistency condition:** the negation may not be derivable.
  - $\gamma$ : **Consequence:** will be concluded.
- A default rule is **closed** if it does not contain free variables.
- **(Closed) default theory:** A pair  $(D, W)$ , where  $D$  is a countable set of (closed) default rules and  $W$  is a countable set of FOL formulae.

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# Extensions of Default Theories

Default theories **extend** the theory given by  $W$  using the default rules  $D$  ( $\rightsquigarrow$  **extensions**). There may be zero, one, or many extensions.

## Example

$$W = \{a, \neg b \vee \neg c\}$$
$$D = \left\{ \frac{a: b}{b}, \frac{a: c}{c} \right\}$$

One **extension** contains  $b$ , the other contains  $c$ .

**Intuitively:** an **extension** is a set of **beliefs** resulting from  $W$  and  $D$ .

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# Decision Problems about Extensions in Default Logic

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**Existence of extensions:** Does a default theory have an extension?

**Credulous reasoning:** If  $\varphi$  is in at least one extension,  $\varphi$  is a credulous default conclusion.

**Skeptical reasoning:** If  $\varphi$  is in all extensions,  $\varphi$  is a skeptical default conclusion.

# Decision Problems about Extensions in Default Logic

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**Existence of extensions:** Does a default theory have an extension?

**Credulous reasoning:** If  $\varphi$  is in at least one extension,  $\varphi$  is a **credulous default conclusion**.

**Skeptical reasoning:** If  $\varphi$  is in all extensions,  $\varphi$  is a **skeptical default conclusion**.



# Extensions – Informally

Desirable properties of an **extension**  $E$  of  $(D, W)$ :

- 1 Contains all facts  $W \subseteq E$ .
- 2 Is deductively closed:  $E = \text{Th}(E)$ .
- 3 All applicable default rules have been applied:

**If**

- 1  $(\frac{\alpha:\beta}{\gamma}) \in D$ ,
- 2  $\alpha \in E$ ,
- 3  $\neg\beta \notin E$

**then**  $\gamma \in E$ .

⇒ Requirement: Application of default rules must follow in sequence (*groundedness*).

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$\Rightarrow$  Requirement: Application of default rules must follow in sequence (*groundedness*).

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## Example

$$\begin{aligned}W &= \emptyset \\D &= \left\{ \frac{a:b}{b}, \frac{b:a}{a} \right\}\end{aligned}$$

*Question:* Should  $\text{Th}(\{a, b\})$  be an extension?

*Answer:* No!

*a* can only be derived if we already have derived *b*.

*b* can only be derived if we already have derived *a*.

## Example

$$\begin{aligned}W &= \emptyset \\D &= \left\{ \frac{a:b}{b}, \frac{b:a}{a} \right\}\end{aligned}$$

*Question:* Should  $\text{Th}(\{a, b\})$  be an extension?

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# Extensions – Formally

## Definition

Let  $\Delta = (D, W)$  be a closed default theory and let  $E$  be a set of closed formulae.

Let

$$E_0 = W$$

$$E_i = \text{Th}(E_{i-1}) \cup \left\{ \gamma \mid \frac{\alpha: \beta}{\gamma} \in D, \alpha \in E_{i-1}, \neg\beta \notin E \right\}$$

Then  $E$  is an **extension** of  $\Delta$  iff

$$E = \bigcup_{i=0}^{\infty} E_i.$$

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# How to Use This Definition?

- The definition does not tell us how to **construct** an extension.
- However, it tells us how to **check** whether a set is an extension.
- Guess a set  $E$ .
- Then construct sets  $E_i$  by starting with  $W$ .
- If  $E = \bigcup_{i=0}^{\infty} E_i$ , then  $E$  is an **extension** of  $(D, W)$ .

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# Examples

$$D = \left\{ \frac{a:b}{b}, \frac{b:a}{a} \right\}$$

$$W = \{a \vee b\}$$

$$D = \left\{ \frac{a:b}{\neg b} \right\}$$

$$W = \emptyset$$

$$D = \left\{ \frac{a:b}{\neg b} \right\}$$

$$W = \{a\}$$

$$D = \left\{ \frac{:a}{a}, \frac{:b}{b}, \frac{:c}{c} \right\}$$

$$W = \{b \rightarrow \neg a \wedge \neg c\}$$

$$D = \left\{ \frac{:c}{\neg d}, \frac{:d}{\neg e}, \frac{:e}{\neg f} \right\}$$

$$W = \emptyset$$

$$D = \left\{ \frac{:c}{\neg d}, \frac{:d}{\neg c} \right\}$$

$$W = \emptyset$$

$$D = \left\{ \frac{a:b}{c}, \frac{a:d}{e} \right\}$$

$$W = \{a, \neg b \vee \neg d\}$$

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# Questions, Questions, Questions ...

- What can we say about the **existence** of extensions?
- How are the different extensions **related** to each other?
  - Can one extension be a **subset** of another one?
  - Are extensions **pairwise incompatible** (i.e. jointly inconsistent)?
- Can an extension be **inconsistent**?

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# Properties of Extensions

## Theorem

- 1 If  $W$  is inconsistent, there is only one extension.
- 2 A closed default theory  $(D, W)$  has an inconsistent extension iff  $W$  is inconsistent.

## Proof idea.

- 1 If  $W$  is inconsistent, no default rule is applicable and  $\text{Th}(W)$  is the only extension.
- 2 Claim 1  $\implies$  the *if*-part.

For *only if*: If  $W$  is consistent, there is a consistent  $E_i$  s.t.  $E_{i+1}$  is inconsistent.

Let  $\{\gamma_1, \dots, \gamma_n\} = E_{i+1} \setminus \text{Th}(E_i)$  (the conclusions of applied defaults). Now  $\{\neg\beta_1, \dots, \neg\beta_n\} \cap E = \emptyset$  because otherwise the defaults are not applicable.

But this contradicts the inconsistency of  $E$ .

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# Properties of Extensions

## Theorem

If  $E$  and  $F$  are extensions of  $(D, W)$  such that  $E \subseteq F$ , then  $E = F$ .

## Proof sketch.

$E = \bigcup_{i=0}^{\infty} E_i$  and  $F = \bigcup_{i=0}^{\infty} F_i$ . Use induction to show  $F_i \subseteq E_i$ .

Base case  $i = 0$ : Trivially  $E_0 = F_0 = W$ .

Inductive case  $i \geq 1$ : Assume  $\gamma \in F_{i+1}$ . Two cases:

- 1  $\gamma \in \text{Th}(F_i)$  implies  $\gamma \in \text{Th}(E_i)$  (because  $F_i \subseteq E_i$  by IH), and therefore  $\gamma \in E_{i+1}$ .
- 2 Otherwise  $\frac{\alpha:\beta}{\gamma} \in D$ ,  $\alpha \in F_i$ ,  $\neg\beta \notin F$ . However, then we have  $\alpha \in E_i$  (because  $F_i \subseteq E_i$ ) and  $\neg\beta \notin E$  (because of  $E \subseteq F$ ), i.e.,  $\gamma \in E_{i+1}$ .



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# Normal Default Theories

All defaults in a **normal default theory** are **normal**:

$$\frac{\alpha : \beta}{\beta}.$$

## Theorem

*Normal default theories have at least one extension.*

## Proof sketch.

If  $W$  inconsistent, trivial. Otherwise construct

$$\begin{aligned} E_0 &= W \\ E_{i+1} &= \text{Th}(E_i) \cup T_i \end{aligned} \quad E = \bigcup_{i=0}^{\infty} E_i$$

where  $T_i$  is a maximal set s.t. (1)  $E_i \cup T_i$  is consistent and (2) if  $\beta \in T_i$  then there is  $\frac{\alpha : \beta}{\beta} \in D$  and  $\alpha \in E_i$ .

Show:  $T_i = \left\{ \beta \mid \frac{\alpha : \beta}{\beta} \in D, \alpha \in E_i, \neg\beta \notin E \right\}$  for all  $i \geq 0$ . □

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# Normal Default Theories: Extensions are Orthogonal

## Theorem (Orthogonality)

*Let  $E$  and  $F$  be distinct extensions of a normal default theory.  
Then  $E \cup F$  is inconsistent.*

## Proof.

Let  $E = \bigcup E_i$  and  $F = \bigcup F_i$  with

$$E_{i+1} = \text{Th}(E_i) \cup \left\{ \beta \mid \frac{\alpha: \beta}{\beta} \in D, \alpha \in E_i, \neg\beta \notin E \right\}$$

and the same for  $F$ . Since  $E \neq F$ , there exists a smallest  $i$  such that  $E_{i+1} \neq F_{i+1}$ . This means there exists  $\frac{\alpha: \beta}{\beta} \in D$  with  $\alpha \in E_i = F_i$  but  $\beta \in E_{i+1}$  and  $\beta \notin F_{i+1}$ . This is only possible if  $\neg\beta \in F$ . This means  $\beta \in E$  and  $\neg\beta \in F$ , i.e.,  $E \cup F$  is inconsistent.  $\square$

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# Default Proofs in Normal Default Theories

## Definition

A **default proof** of  $\gamma$  in a normal default theory  $(D, W)$  is a finite sequence of defaults  $(\delta_i = \frac{\alpha_i \cdot \beta_i}{\beta_i})_{i=1, \dots, n}$  such that

- 1  $W \cup \{\beta_1, \dots, \beta_n\} \vdash \gamma$ ,
- 2  $W \cup \{\beta_1, \dots, \beta_n\}$  is consistent, and
- 3  $W \cup \{\beta_1, \dots, \beta_k\} \vdash \alpha_{k+1}$ , for  $0 \leq k \leq n - 1$ .

## Theorem

*Let  $\Delta = \langle D, W \rangle$  be a normal default theory so that  $W$  is consistent. Then  $\gamma$  has a default proof in  $\Delta$  iff there exists an extension  $E$  of  $\Delta$  such that  $\gamma \in E$ .*

Test 2 (**consistency**) in the proof procedure suggests that default provability is not even **semi-decidable**.

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## Theorem

*It is not semi-decidable to test whether a formula follows (skeptically or credulously) from a default theory.*

## Proof.

Let  $(D, W)$  be a default theory with  $W = \emptyset$  and  $D = \left\{ \frac{\cdot}{\beta} \right\}$  with  $\beta$  an arbitrary closed FOL formula. Clearly,  $\beta$  is in some/all extensions of  $(D, W)$  if and only if  $\beta$  is satisfiable.

The existence of a semi-decision procedure for default proofs implies that there is a semi-decision procedure for satisfiability in FOL.

But this is not possible because FOL validity is semi-decidable and this together with semi-decidability of FOL satisfiability would imply decidability of FOL, which is not the case.  $\square$

# Propositional Default Logic

- Propositional DL is decidable.
- How difficult is reasoning in propositional DL?
- The skeptical default reasoning problem (does  $\varphi$  follow from  $\Delta$  skeptically:  $\Delta \vdash \sim \varphi$ ?) is called PDS, credulous reasoning is called LPDS.
- (L)PDS is co-NP-hard (let  $D = \emptyset$ ,  $W = \emptyset$ ) and NP-hard (let  $W = \emptyset$ ,  $D = \left\{ \frac{:\beta}{\beta} \right\}$ ).

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# Complexity of DL – Outline

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# Skeptical Reasoning in Propositional DL

## Lemma

$PDS \in \Pi_2^p$ .

## Proof.

We show that the complementary problem **UNPDS** (is there an extension  $E$  such that  $\varphi \notin E$ ) is in  $\Sigma_2^p$ .

The **algorithm**: **Guess** set  $T \subseteq D$  of defaults: those that are applied.

**Verify** that defaults in  $T$  lead to  $E$ , using a **SAT oracle** and the guessed  $E = \text{Th} \left( \left\{ \gamma \mid \frac{\alpha:\beta}{\gamma} \in T \right\} \cup W \right)$ .

**Verify** that  $\left\{ \gamma \mid \frac{\alpha:\beta}{\gamma} \in T \right\} \cup W \not\models \varphi$  (**SAT oracle**).

$\rightsquigarrow$  UNPDS  $\in \Sigma_2^p$ . □

**Note**: LPDS  $\in \Sigma_2^p$ .

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# $\Pi_2^p$ -Hardness

## Lemma

*PDS is  $\Pi_2^p$ -hard.*

## Proof.

**Reduction from 2QBF to UNPDS:** For  $\exists \vec{a} \forall \vec{b} \phi(\vec{a}, \vec{b})$  with  $\vec{a} = a_1, \dots, a_n$  and  $\vec{b} = b_1, \dots, b_m$  construct  $\Delta = (D, W)$  with

$$D = \left\{ \frac{:a_i}{a_i}, \frac{:\neg a_i}{\neg a_i}, \frac{:\neg \phi(\vec{a}, \vec{b})}{\neg \phi(\vec{a}, \vec{b})} \right\}, \quad W = \emptyset$$

No extension contains both  $a_i$  and  $\neg a_i$ .

Now

$\Delta \not\models \neg \phi(\vec{a}, \vec{b})$  iff there is extension  $E$  s.t.  $\neg \phi(\vec{a}, \vec{b}) \notin E$   
iff there is  $E$  s.t.  $\phi(\vec{a}, \vec{b}) \in E$  (by  $\frac{:\neg \phi(\vec{a}, \vec{b})}{\neg \phi(\vec{a}, \vec{b})} \in D$ )  
iff there is  $A \subseteq \{a_1, \neg a_1, \dots, a_n, \neg a_n\}$  s.t.  $A \models \phi(\vec{a}, \vec{b})$   
iff  $\exists \vec{a} \forall \vec{b} \phi(\vec{a}, \vec{b})$  is true.

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No extension contains both  $a_i$  and  $\neg a_i$ .

Now

$\Delta \models \neg \phi(\vec{a}, \vec{b})$  iff there is extension  $E$  s.t.  $\neg \phi(\vec{a}, \vec{b}) \in E$   
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# $\Pi_2^p$ -Hardness

## Lemma

*PDS is  $\Pi_2^p$ -hard.*

## Proof.

Reduction from 2QBF to UNPDS: For  $\exists \vec{a} \forall \vec{b} \phi(\vec{a}, \vec{b})$  with  $\vec{a} = a_1, \dots, a_n$  and  $\vec{b} = b_1, \dots, b_m$  construct  $\Delta = (D, W)$  with

$$D = \left\{ \frac{:a_i}{a_i}, \frac{:\neg a_i}{\neg a_i}, \frac{:\neg \phi(\vec{a}, \vec{b})}{\neg \phi(\vec{a}, \vec{b})} \right\}, \quad W = \emptyset$$

No extension contains both  $a_i$  and  $\neg a_i$ .

Now

$\Delta \not\models \neg \phi(\vec{a}, \vec{b})$  iff there is extension  $E$  s.t.  $\neg \phi(\vec{a}, \vec{b}) \notin E$

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iff  $\exists \vec{a} \forall \vec{b} \phi(\vec{a}, \vec{b})$  is true.

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# Conclusions & Remarks

## Theorem

*PDS is  $\Pi_2^p$ -complete, even for defaults of the form  $\frac{: \alpha}{\alpha}$ .*

## Theorem

*LPDS is  $\Sigma_2^p$ -complete, even for defaults of the form  $\frac{: \alpha}{\alpha}$ .*

- PDS is “easier” than reasoning in most modal logics.
- General and normal defaults have the same complexity.
- Polynomial special cases cannot be achieved by restricting, for example, to **Horn clauses** (satisfiability testing in polynomial time).
- It is necessary to restrict the underlying **monotonic reasoning problem** and the **number of extensions**.
- Similar results hold for other **nonmonotonic logics**.

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# Semi-Normal Defaults (1)

Semi-normal defaults are sometimes useful:

$$\frac{\alpha : \beta \wedge \gamma}{\beta}$$

Important when one has **interacting** defaults:

$$\frac{\text{Adult}(x) : \text{Employed}(x)}{\text{Employed}(x)}$$

$$\frac{\text{Student}(x) : \text{Adult}(x)}{\text{Adult}(x)}$$

$$\frac{\text{Student}(x) : \neg\text{Employed}(x)}{\neg\text{Employed}(x)}$$

For **Student(TOM)** we get two extensions: one with **Employed(Tom)** and the other one with  $\neg\text{Employed}(\text{Tom})$ . Since the third rule is “**more specific**”, we may prefer it.

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## Semi-Normal Defaults (2)

- Since being a student is an exception, we could use a **semi-normal** default to exclude students from employed adults:

$$\frac{\text{Student}(x) : \neg\text{Employed}(x)}{\neg\text{Employed}(x)}$$
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$$\frac{\text{Student}(x) : \text{Adult}(x)}{\text{Adult}(x)}$$

- Representing conflict-resolution by semi-normal defaults becomes clumsy when the number of default rules becomes high.
- A scheme for assigning **priorities** would be more elegant (there are indeed such schemes).

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# Open Defaults (1)

- Our examples included **open defaults**, but the theory covers only **closed defaults**.
- If we have  $\frac{\alpha(\vec{x}):\beta(\vec{x})}{\gamma(\vec{x})}$ , then the variables should stand for all **nameable** objects.
- **Problem**: What about objects that have been introduced implicitly:  $\boxed{\exists x P(x)}$ .
- **Solution by Reiter**: Skolemization of all formulae in  $W$  and  $D$ .
- **Interpretation**: An open default stands for all the closed defaults resulting from substituting **ground terms** for the variables.

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## Open Defaults (2)

Skolemization can create problems because it preserves satisfiability, but it is not an equivalence transformation.

### Example

$$\forall x(\text{Man}(x) \leftrightarrow \neg \text{Woman}(x))$$
$$\forall x(\text{Man}(x) \rightarrow (\exists y(\text{Spouse}(x, y) \wedge \text{Woman}(y)) \vee \text{Bachelor}(x)))$$
$$\text{Man}(\text{TOM})$$
$$\text{Spouse}(\text{TOM}, \text{MARY})$$
$$\text{Woman}(\text{MARY})$$
$$\frac{: \text{Man}(x)}{\text{Man}(x)}$$

Skolemization of  $\exists y$ : ... enables concluding **Bachelor(TOM)**!  
The reason is that for  $g(\text{TOM})$  we get  $\text{Man}(g(\text{TOM}))$  by default  
( $g$  is the Skolem function).

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## Open Defaults (3)

It is even worse: Logically equivalent theories can have different extensions:

$$W_1 = \{\exists x(P(C, x) \vee Q(C, x))\}$$

$$W_2 = \{\exists xP(C, x) \vee \exists xQ(C, x)\}$$

$$D = \left\{ \frac{P(x, y) \vee Q(x, y) : R}{R} \right\}$$

$W_1$  and  $W_2$  are logically equivalent. However, the Skolemization of  $W_1$ , symbolically  $s(W_1)$ , is not equivalent with  $s(W_2)$ . The only extension of  $(D, W_1)$  is  $\text{Th}(s(W_1) \cup R)$ . The only extension of  $(D, W_2)$  is  $\text{Th}(s(W_2))$ .

**Note:** Skolemization is not the right method to deal with open defaults in the general case.

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Although Reiter's definition of DL makes sense, one can come up with a number of variations and extend the investigation ...

- Extensions can be defined differently (e.g., by remembering consistency conditions).
- ... or by removing the groundedness condition.
- Open defaults can be handled differently (more model-theoretically).
- General proof methods for the finite, decidable case
- Applications of default logic:
  - Diagnosis
  - Reasoning about actions

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