Principles of Knowledge Representation and Reasoning

Propositional Logic

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Forms

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Why Logic?

- Logic is one of the best developed systems for representing knowledge.
- Can be used for analysis, design and specification.
- Understanding formal logic is a prerequisite for understanding most research papers in KRR.

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Forms

Decision Problems

- Logics of different orders (1st, 2nd, ...)
- Modal logics
 - epistemic
 - temporal
 - dynamic (program)
 - multi-modal logics
 - . . .
- Many-valued logics
- Conditional logics
- Nonmonotonic logics
- Linear logics
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Why Logic?

Propositiona Logic

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Semantics

Terminology

Normal Form

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Why Logic?

Propositiona Logic

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Semantics

Terminology

Normal Forms

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Why Logic?

Propositiona Logic

Semantics

Terminology

Normal Forms

Decision

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Why Logic?

Propositiona Logic

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Semantics

Terminology

Normal Forms

Decision Problems

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Why Logic?

Propositiona Logic

Terminology

Normal Form

Decision Problems

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Why Logic?

Propositiona Logic

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Semantics

Terminology

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Why Logic?

Propositiona Logic

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Semantics

Terminology

Normal Form

Decision Problems

Resolution

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Why Logic?

Propositiona Logic

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Semantics

Terminology

Normal Forms

Decision Problems

- Define a formal language: logical & non-logical symbols, syntax rules
- Provide language with compositional semantics
 - Fix universe of discourse
 - Specify how the non-logical symbols can be interpreted: interpretation
 - Rules how to combine interpretation of single symbols
 - Satisfying interpretation = model
 - Semantics often entails concept of logical implication/entailment
- Specify a calculus that allows to derive new formulae from old ones – according to the entailment relation

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Why Logic?

Propositional Logic

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Why Logic?

Propositional Logic

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Propositional Logic

Terminology

Normal Forms

Problems

Propositional Logic: Main Ideas

- Non-logical symbols: propositional variables or atoms
 - representing propositions which cannot be decomposed
 - which can be true or false (for example: "Snow is white", "It rains")
- Logical symbols: propositional connectives such as: and (∧), or (∨), and not (¬)
- Formulae: built out of atoms and connectives
- Universe of discourse: truth values

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Propositional Logic

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Semantics

Terminology

Normal Forms

Decision Problems

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

Decision Problems

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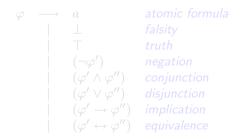
Semantics

Terminology

Normal Forms

Problems

Countable alphabet Σ of atomic propositions: a,b,c,\ldots Propositional formulae are built according to the following rule:



Parentheses can be omitted if no ambiguity arises

Operator precedence:
$$\neg > \land > \lor > \rightarrow = \leftrightarrow$$
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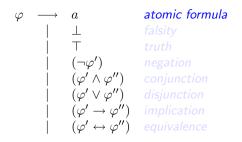
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Terminology

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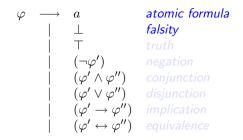
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Propositional Logic

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Propositional Logic

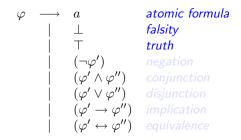
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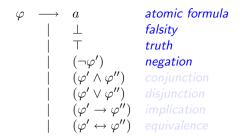
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Propositiona Logic

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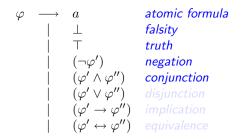
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Propositional Logic

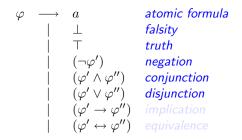
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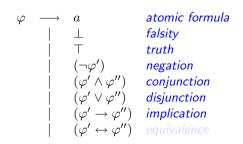
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Syntax

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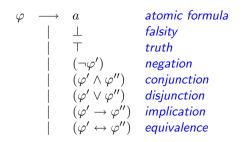
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Propositional Logic

Syntax

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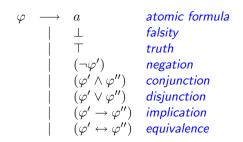
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Propositiona Logic

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Why Logic?

Propositiona Logic

Syntax

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Decision

- Atomic propositions can be true (1,T) or false (0,F).
- Provided the truth values of the atoms have been fixed (truth assignment or interpretation), the truth value of a formula can be computed from the truth values of the atoms and the connectives.
- Example:

$$(a \lor b) \land c$$

is true iff c is true and, additionally, a or b is true.

Logical implication can then be defined as follows:

• φ is implied by a set of formulae Θ iff φ is true for all truth assignments (world states) that make all formulae in Θ true.

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

Decision

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

Decision Problems

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

Problems

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

Decision Problems

Formal Semantics

An interpretation or truth assignment over Σ is a function:

$$\mathcal{I} \colon \Sigma \to \{T, F\}.$$

A formula ψ is true under \mathcal{I} or is satisfied by \mathcal{I} (symb. $\mathcal{I} \models \psi$):

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Why Logic?

Propositional Logic

Semantics

Scillaticies

Terminology

Normal Forms

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A formula ψ is true under \mathcal{I} or is satisfied by \mathcal{I} (symb. $\mathcal{I} \models \psi$):

$$\begin{split} \mathcal{I} &\models a & \text{iff} & \mathcal{I}(a) = T \\ & \qquad \qquad \mathcal{I} \models \top \\ & \qquad \qquad \mathcal{I} \not\models \bot \\ \\ \mathcal{I} &\models \neg \varphi & \text{iff} & \qquad \mathcal{I} \not\models \varphi \\ \\ \mathcal{I} &\models \varphi \land \varphi' & \text{iff} & \qquad \mathcal{I} \models \varphi \text{ and } \mathcal{I} \models \varphi' \\ \\ \mathcal{I} &\models \varphi \lor \varphi' & \text{iff} & \qquad \mathcal{I} \models \varphi \text{ or } \mathcal{I} \models \varphi' \\ \\ \mathcal{I} &\models \varphi \to \varphi' & \text{iff} & \qquad \mathcal{I} \models \varphi, \text{ then } \mathcal{I} \models \varphi' \\ \\ \mathcal{I} &\models \varphi \leftrightarrow \varphi' & \text{iff} & \qquad \mathcal{I} \models \varphi \text{ if and only if } \mathcal{I} \models \varphi' \\ \end{split}$$

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Propositional Logic

Semantics

Terminology

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Resolution

Example

Given

$$\mathcal{I}: a \mapsto \mathbf{T}, b \mapsto \mathbf{F}, c \mapsto \mathbf{F}, d \mapsto \mathbf{T},$$

Is
$$((a \lor b) \leftrightarrow (c \lor d)) \land (\neg(a \land c) \lor (c \land \neg d))$$
 true or false?

$$((\mathbf{a} \lor \mathbf{b}) \leftrightarrow (\mathbf{c} \lor \mathbf{d})) \land (\neg(\mathbf{a} \land \mathbf{c}) \lor (\mathbf{c} \land \neg \mathbf{d}))$$

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Why Logic?

Propositional Logic

Semantics

Torminology

Decision

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Why Logic?

Propositional Logic

Semantics

Torminology

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Decision

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Forms

Problems

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Forms

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Why Logic?

Propositional Logic

Semantics

reminology

Normal Forms

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Why Logic?

Propositional Logic

Semantics

Terminology

reminology

Decision

An interpretation ${\mathcal I}$ is a model of φ iff

$$\mathcal{I} \models \varphi$$

A formula φ is

- satisfiable if there is an \mathcal{I} such that $\mathcal{I} \models \varphi$;
- unsatisfiable, otherwise; and
- valid if $\mathcal{I} \models \varphi$ for each \mathcal{I} ;
- falsifiable, otherwise.

Two formulae φ and ψ are logically equivalent (symb. $\varphi \equiv \psi$) if for all interpretations \mathcal{I} ,

$$\mathcal{I} \models \varphi \text{ iff } \mathcal{I} \models \psi.$$

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Why Logic?

Propositional Logic

Semantics

Terminology

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Why Logic?

Propositional Logic

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Semantics

Terminology

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$$\mathcal{I} \models \varphi$$

A formula φ is

- satisfiable if there is an \mathcal{I} such that $\mathcal{I} \models \varphi$;
- unsatisfiable, otherwise; and
- valid if $\mathcal{I} \models \varphi$ for each \mathcal{I} ;
- falsifiable, otherwise.

Two formulae φ and ψ are logically equivalent (symb. $\varphi \equiv \psi$) if for all interpretations \mathcal{I} ,

$$\mathcal{I} \models \varphi \text{ iff } \mathcal{I} \models \psi.$$

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Why Logic?

Propositional Logic

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Semantics

Terminology

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Why Logic?

Propositional Logic

Semantics

Terminology

ecision

Satisfiable, unsatisfiable, falsifiable, valid? $(a \lor b \lor \neg c) \land (\neg a \lor \neg b \lor d) \land (\neg a \lor b \lor \neg d)$

- \rightsquigarrow satisfiable: $a \mapsto T, b \mapsto F, d \mapsto F, \dots$
- \rightsquigarrow falsifiable: $a \mapsto F, b \mapsto F, c \mapsto T, \dots$

$$((\neg a \to \neg b) \to (b \to a))$$

- \rightarrow satisfiable: $a \mapsto T, b \mapsto T$
- valid: Consider all interpretations or argue about falsifying ones.

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

Decision

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Equivalence?
$$\neg(a \lor b) \equiv \neg a \land \neg b$$

Satisfiable, unsatisfiable, falsifiable, valid?

$$(a \vee b \vee \neg c) \wedge (\neg a \vee \neg b \vee d) \wedge (\neg a \vee b \vee \neg d)$$

$$\rightsquigarrow$$
 satisfiable: $a \mapsto T, b \mapsto F, d \mapsto F, \dots$

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

Decision

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

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Why Logic?

Propositional Logic

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Terminology

Normal Forms

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Why Logic?

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Why Logic?

Propositional Logic

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Some Obvious Consequences

Proposition

 φ is valid iff $\neg \varphi$ is unsatisfiable and φ is satisfiable iff $\neg \varphi$ is falsifiable.

Proposition

 $\varphi \equiv \psi \text{ iff } \varphi \leftrightarrow \psi \text{ is valid.}$

Theorem

If $\varphi \equiv \psi$ and χ' results from substituting φ by ψ in χ , then $\chi' \equiv \chi$.

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

Problems

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Why Logic?

Propositional Logic

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Semantics

Terminology

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Decision Problems

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Why Logic?

Propositional Logic

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Semantics

Terminology

Iormal Forms

Problems

Some Equivalences

simplifications	$\varphi o \psi$	\equiv	$\neg \varphi \lor \psi$	$\varphi \leftrightarrow \psi$	\equiv	$(\varphi \to \psi) \land (\psi \to \varphi)$
idempotency	$\varphi \lor \varphi$	\equiv	φ	$\varphi \wedge \varphi$	\equiv	φ
commutativity	$\varphi \lor \psi$	\equiv	$\psi \lor \varphi$	$\varphi \wedge \psi$	\equiv	$\psi \wedge \varphi$
associativity	$(\varphi \lor \psi) \lor \chi$	\equiv	$\varphi \lor (\psi \lor \chi)$	$(\varphi \wedge \psi) \wedge \chi$	\equiv	$\varphi \wedge (\psi \wedge \chi)$
absorption	$\varphi \lor (\varphi \land \psi)$	\equiv	φ	$\varphi \wedge (\varphi \vee \psi)$	\equiv	φ
distributivity	$\varphi \wedge (\psi \vee \chi)$	\equiv	$(\varphi \wedge \psi) \vee$	$\varphi \lor (\psi \land \chi)$	\equiv	$(\varphi \lor \psi) \land$
			$(\varphi \wedge \chi)$			$(\varphi \vee \chi)$
double negation	$\neg \neg \varphi$	\equiv	φ			
constants	$\neg \top$	\equiv	\perp	$\neg \bot$	\equiv	Т
De Morgan	$\neg(\varphi \lor \psi)$	\equiv	$\neg \varphi \wedge \neg \psi$	$\neg(\varphi \wedge \psi)$	\equiv	$\neg \varphi \lor \neg \psi$
truth	$\varphi \lor \top$	\equiv	Τ	$\varphi \wedge \top$	\equiv	φ
falsity	$\varphi \lor \bot$	\equiv	φ	$\varphi \wedge \bot$	\equiv	
taut./contrad.	$\varphi \vee \neg \varphi$	\equiv	Т	$\varphi \wedge \neg \varphi$	\equiv	\perp

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Why Logic?

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Semantics

Terminology

Normal Forms

Decision

... for a given *finite* alphabet Σ ?

- Infinitely many: $a, a \lor a, a \land a, a \lor a \lor a, \dots$
- How many different logically distinguishable (not equivalent) formulae?
 - For Σ with $n = |\Sigma|$, there are 2^n different interpretations.
 - A formula can be characterized by its set of models (if two formulae are not logically equivalent, then their sets of models differ).
 - There are $2^{(2^n)}$ different sets of interpretations.
 - There are $2^{(2^n)}$ (logical) equivalence classes of formulae.

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Why Logic?

Propositional Logic

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Semantics

 ${\sf Terminology}$

Normal Forms

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Why Logic?

Propositional Logic

Syllean

Semantics

 ${\sf Terminology}$

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Why Logic?

Propositiona Logic

Syllean

Semantics

Terminology

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Why Logic?

Propositiona Logic

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Semantics

Terminology

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Why Logic?

Propositional Logic

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Semantics

Terminology

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Why Logic?

Propositional Logic

Sylleax

Semantics

Terminology

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Why Logic?

Propositional Logic

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Terminology

Terminology

Decision

• Extension of the relation \models to sets Θ of formulae:

$$\mathcal{I} \models \Theta \text{ iff } \mathcal{I} \models \varphi \text{ for all } \varphi \in \Theta.$$

• φ is logically implied by Θ (symbolically $\Theta \models \varphi$) iff φ is true in all models of Θ :

$$\Theta \models \varphi \text{ iff } \mathcal{I} \models \varphi \text{ for all } \mathcal{I} \text{ such that } \mathcal{I} \models \Theta$$

- Some consequences:
 - Deduction theorem: $\Theta \cup \{\varphi\} \models \psi \text{ iff } \Theta \models \varphi \rightarrow \psi$
 - Contraposition: $\Theta \cup \{\varphi\} \models \neg \psi \text{ iff } \Theta \cup \{\psi\} \models \neg \varphi$
 - Contradiction: $\Theta \cup \{\varphi\}$ is unsatisfiable iff $\Theta \models \neg \varphi$

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Forms

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Why Logic?

Propositional Logic

Semantics

Terminology

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Forms

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

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 ${\sf Resolution}$

• Extension of the relation \models to sets Θ of formulae:

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Why Logic?

Propositional Logic

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Semantics

Terminology

Iormal Forms

Problems

Terminology:

- Atomic formulae a, negated atomic formulae $\neg a$, truth \top and falsity \bot are literals.
- A disjunction of literals is a clause.
- If ¬ only occurs in front of an atom and there are no occurrences of → and ↔, the formula is in negation normal form (NNF).

Example: $(\neg a \lor \neg b) \land c$, but not: $\neg (a \land b) \land c$

 A conjunction of clauses is in conjunctive normal form (CNF).

Example: $(a \lor b) \land (\neg a \lor c)$

 The dual form (disjunction of conjunctions of literals) is in disjunctive normal form (DNF). KRR

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Why Logic?

Propositional Logic

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Jemantics

Terminology

Normal Forms

Problems

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Why Logic?

Propositional Logic

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Normal Forms

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Why Logic?

Propositional Logic

Syllean

Semantics

Terminology

Normal Forms

Decision

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• The dual form (disjunction of conjunctions of literals) is in disjunctive normal form (DNF). Example: $(a \wedge b) \vee (\neg a \wedge c)$

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Why Logic?

Propositional Logic

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Normal Forms

Decision

Negation Normal Form

Theorem

For each propositional formula there is a logically equivalent formula in NNF.

Proof

First eliminate \rightarrow and \leftrightarrow by the appropriate equivalences. The res of the proof is by structural induction.

Base case: Claim is true for a, $\neg a$, \top , \bot .

Inductive case: Assume claim is true for all formulae φ (up to a certain number of connectives) and call its NNF $\mathsf{nnf}(\varphi)$.

- $\bullet \ \operatorname{nnf}(\varphi \wedge \psi) = \operatorname{nnf}(\varphi) \wedge \operatorname{nnf}(\psi)$
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- $\bullet \ \operatorname{nnf}(\neg(\varphi \wedge \psi)) = \operatorname{nnf}(\neg\varphi) \vee \operatorname{nnf}(\neg\psi)$
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Why Logic?

Propositional Logic

Terminology

Normal Forms

Decision

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Why Logic?

Propositional Logic

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Terminology

Normal Forms

Decision

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Why Logic?

Propositional Logic

Terminology

Normal Forms

Decision

Theorem

For each propositional formula there is a logically equivalent formula in NNF.

Proof.

First eliminate \to and \leftrightarrow by the appropriate equivalences. The rest of the proof is by structural induction.

Base case: Claim is true for a, $\neg a$, \top , \bot .

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KRR

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Why Logic?

Propositional Logic

reminology

Normal Forms

Problems

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Forms

Decision Problems

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KRR

Nebel, Wölfl Ragni

Why Logic?

Propositional Logic

Normal Forms

Decision

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Why Logic?

Propositional Logic

Terminology

Normal Forms

Decision

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Why Logic?

Propositional Logic

Terminology

Normal Forms

Decision

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The claim is true for $a, \neg a, \top, \bot$.

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- Assume $\mathrm{cnf}(\varphi)=\bigwedge_i\chi_i$ and $\mathrm{cnf}(\psi)=\bigwedge_j\rho_j$ with χ_i,ρ_j being clauses. Then

$$\begin{split} \operatorname{cnf}(\varphi \vee \psi) &= & \operatorname{cnf}((\bigwedge_i \chi_i) \vee (\bigwedge_j \rho_j)) \\ &= & \bigwedge_i (\chi_i \vee \rho_j) \quad \text{(by distributivity)} \end{split}$$

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Why Logic?

Propositional Logic

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Terminology

Normal Forms

Decision

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KRR

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Why Logic?

Propositiona Logic

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Terminology

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Normal Forms
Decision

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KRR

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Why Logic?

Propositional Logic

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Normal Forms

Decision

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KRR

Nebel, Wölfl, Ragni

Why Logic?

Propositional Logic

- 3

Terminology

Normal Forms
Decision

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KRR

Nebel, Wölfl, Ragni

Why Logic?

Propositional Logic

Torminology

Normal Forms

Decision

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Nebel, Wölfl, Ragni

Why Logic?

Propositional Logic

Terminology

Normal Forms

Decision

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Why Logic?

Propositiona Logic

Terminology

Normal Forms

Decision

How do we decide whether a formula is satisfiable, unsatisfiable, valid, or falsifiable?

Note: Satisfiability and falsifiability are NP-complete. Validity and unsatisfiability are co-NP-complete.

- A CNF formula is valid iff all clauses contain two complementary literals or ⊤.
- A DNF formula is satisfiable iff one disjunct does not contain ⊥ or two complementary literals.
- However, transformation to CNF or DNF may take exponential time (and space!).
- One can try out all truth assignments.
- One can test systematically for satisfying truth assignments (backtracking search)
 Davis-Putnam-Logemann-Loveland procedure (DPLL)

KRR

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Why Logic?

Propositional Logic

-,..-..

Semantics

Terminology

Normal Forms

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KRR

Nebel, Wölfl, Ragni

Why Logic?

Propositiona Logic

-,..-..

Semantics

Terminology

Normal Forms

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KRR

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Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Forms

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KRR

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Why Logic?

Propositiona Logic

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Semantics

Terminology

Normal Forms

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KRR

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Why Logic?

Propositiona Logic

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Semantics

Terminology

Normal Forms

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Why Logic?

Propositiona Logic

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Semantics

Terminology

lormal Forms

Decision Problems

- We want to decide $\Theta \models \varphi$.
- Use deduction theorem and reduce to validity

$$\Theta \models \varphi \quad \text{iff} \quad \bigwedge \Theta \to \varphi \text{ is valid.}$$

- Now negate and test for unsatisfiability using DPLL.
- Different approach: Try to derive φ from Θ find a proof of φ from Θ .
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Why Logic?

Propositional Logic

Syllean

Semantics

Terminology

Normal Forms

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Why Logic?

Propositional Logic

Jylltax

Semantics

Terminology

Normal Form

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Nebel, Wölfl, Ragni

Why Logic?

Propositional Logic

Jylliax

Semantics

Terminology

Normal Forms

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Why Logic?

Propositional Logic

Jylliax

Semantics

Terminology

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Why Logic?

Propositional Logic

Syllean

Semantics

Terminology

Normal Forms

Decision Problems

Resolution: Representation

- We assume that all formulae are in CNF.
 - Can be generated using the described method.
 - Often formulae are already close to CNF.
 - There is a "cheap" conversion from arbitrary formulae to CNF that preserves satisfiability – which is enough as we will see.
- More convenient representation:
 - CNF formula is represented as a set.
 - Each clause is a set of literals.
 - $\bullet \ (a \vee \neg b) \wedge (\neg a \vee c) \rightsquigarrow \{\{a, \neg b\}, \{\neg a, c\}\}\}$
- Empty clause (symbolically □) and empty set of clauses (symbolically ∅) are different!

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Why Logic?

Propositional Logic

Terminology

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Why Logic?

Propositional Logic

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Terminology

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Why Logic?

Propositional Logic

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Terminology

Normal Form

Decision

Resolution

Let l be a literal and \bar{l} its complement.

The resolution rule

$$\frac{C_1 \dot{\cup} \{l\}, C_2 \dot{\cup} \{\bar{l}\}}{C_1 \cup C_2}$$

 $C_1 \cup C_2$ is the resolvent of the parent clauses $C_1 \cup \{l\}$ and $C_2 \cup \{\bar{l}\}$. l and \bar{l} are the resolution literals.

Example: $\{a, b, \neg c\}$ resolves with $\{a, d, c\}$ to $\{a, b, d\}$.

Note: The resolvent is <u>not</u> logically equivalent to the set of parent clauses!

Notation:

 $R(\Delta) = \{C|C \text{ is resolvent of two clauses in } \Delta\}$

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Why Logic?

Propositional Logic

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Terminology

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KRR

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Why Logic?

Propositional Logic

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Terminology

Normal Forms

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Why Logic?

Propositional Logic

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Terminology

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KRR

Nebel, Wölfl, Ragni

Why Logic?

Propositional Logic

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Semantics

Terminology

Normal Form

Decision

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Example: $\{a, b, \neg c\}$ resolves with $\{a, d, c\}$ to $\{a, b, d\}$.

Note: The resolvent is <u>not</u> logically equivalent to the set of parent clauses!

Notation:

 $R(\Delta) = \{C|C \text{ is resolvent of two clauses in } \Delta\}$

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Forms

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Resolution

Resolution: Derivations

D can be derived from Δ by resolution (symbolically $\Delta \vdash D$) if there is a sequence C_1, \ldots, C_n of clauses such that

- $C_i \in R(\Delta \cup \{C_1, \dots, C_{i-1}\}), \text{ for all } i \in \{1, \dots, n\}.$

Define $R^*(\Delta) = \{D | \Delta \vdash D\}.$

Theorem (Soundness of resolution)

Let D be a clause. If $\Delta \vdash D$ then $\Delta \models D$.

Proof idea

Show $\Delta \models D$ if $D \in R(\Delta)$ and use induction on proof length.

Let $C_1 \cup \{l\}$ and $C_2 \cup \{\bar{l}\}$ be the parent clauses of $D = C_1 \cup C_2$.

Assume $\mathcal{I} \models \Delta$, we have to show $\mathcal{I} \models D$.

Case 1: $\mathcal{I} \models l$ then there must be a literal $m \in C_2$ s.t. $\mathcal{I} \models m$. This implies $\mathcal{I} \models D$.

Case 2: $\mathcal{I} \models \overline{l}$ similarly, there is $m \in C_1$ s.t. $\mathcal{I} \models m$.

This means that each model \mathcal{I} of Δ also satisfies D, i.e., $\Delta \models D$.

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Why Logic?

Propositional Logic

Semantics

Terminology

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Decision Problems

Resolution

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Form

Decision Problems

Resolution

Derivations Completeness Resolution Strategies

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Form

Decision Problems

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Form

Decision Problems

Resolution

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Form

Decision Problems

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Resolution

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Form

Decision Problems

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Resolution

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Why Logic?

Propositional Logic

Semantics

Terminology

Normal Form

Decision Problems

Resolution

Do we have

$$\Delta \models \varphi \text{ implies } \Delta \vdash \varphi?$$

Of course, could only hold for CNF. However

$$\left\{ \{a,b\}, \{\neg b,c\} \right\} \models \left\{ a,b,c \right\} \\ \not\vdash \left\{ a,b,c \right\}$$

However, one can show that resolution is refutation-complete:

 Δ is unsatisfiable iff $\Delta \vdash \Box$.

Entailment: Reduce to unsatisfiability testing and decide by resolution

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Why Logic?

Propositional Logic

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Resolution

Derivations Completeness Resolution Strategies

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Why Logic?

Propositional Logic

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Terminology

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Why Logic?

Propositional Logic

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Derivations
Completeness
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Why Logic?

Propositional Logic

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Why Logic?

Propositional Logic

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Terminology

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Resolution

- Trying out all different resolutions can be very costly,
- and might not be necessary.
- There are different resolution strategies.
- Examples:
 - Input resolution $(R_I(\cdot))$: In each resolution step, one of the parent clauses must be a clause of the input set.
 - Unit resolution $(R_U(\cdot))$: In each resolution step, one of the parent clauses must be a unit clause.
 - Not all strategies are (refutation) completeness preserving.
 Neither input nor unit resolution is. However, there are others.

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Why Logic?

Propositiona Logic

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Semantics

Terminology

Decision

Problems

Derivations Completeness

Completeness Resolution Strategies Horn Clauses

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Why Logic?

Propositiona Logic

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Why Logic?

Propositional Logic

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Terminology

Decision

Resolution

Horn clauses: Clauses with at most one positive literal Example: $(a \lor \neg b \lor \neg c), (\neg b \lor \neg c)$

Proposition

Unit resolution is refutation-complete for Horn clauses

Proof idea

Consider $R_U^*(\Delta)$ of Horn clause set Δ . We have to show that if $\Box \not \in R_U^*(\Delta)$, then $\Delta (\equiv R_U^*(\Delta))$ is satisfiable.

- Assign *true* to all unit clauses in $R_U^*(\Delta)$.
- Those clauses that do not contain a literal l such that $\{l\}$ is one of the unit clauses have at least one negative literal.
- Assign true to these literals.
- Results in satisfying truth assignment for $R_U^*(\Delta)$ (and $\Delta \subseteq R_U^*(\Delta)$).

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Why Logic?

Propositional Logic

Terminology

Normal Forms

Problems

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Why Logic?

Propositional Logic

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Terminology

Normal Form

Decision

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Why Logic?

Propositional Logic

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Terminology

Decision

Problems

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Why Logic?

Propositional Logic

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Normal Forms

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Why Logic?

Propositional Logic

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Decision Problems

Resolution

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Why Logic?

Propositional Logic

Terminology

Normal Form

Decision

Resolution