Foundations of Artificial Intelligence

Prof. Dr. B. Nebel, Prof. Dr. M. Riedmiller
S. Lange, J. Witkowski, D. Zhang
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University of Freiburg
Department of Computer Science

Exercise Sheet 11

Due: Tuesday, July 20, 2010

Exercise 11.1 (Syntax and Semantics of Predicate Logic)

(a) Classify the following expressions as terms, ground terms, atoms, formulae, sentences, or statements in meta language. If there is more than one possibility for an expression please list them all. In the expressions, a and b are constants, x and y are variables, f and g are functions, and P and Q are predicates.

(a) $P(x, y)$
(b) $f(a, b)$
(c) $\mathcal{I} \models P(a, f(b))$
(d) $\mathcal{I}, \alpha \models P(a, f(x))$
(e) $f(g(x), b)$
(f) $Q(x)$ is satisfiable.

(g) $\exists x(P(x, y) \wedge Q(x)) \vee P(y, x)$
(h) $\forall x(\exists y(P(x, y) \wedge Q(x)) \vee P(x, y))$
(i) $\forall x \forall y(P(x, y) \wedge Q(x) \vee P(f(y), x))$
(j) $Q(x) \vee P(x, y) \equiv P(x, y) \vee Q(x)$

(b) Consider the following set of formulae:

$$\Theta = \left\{ \begin{array}{l} \forall x \neg P(x, x) \\ \forall x \forall y \forall z((P(x, y) \wedge P(y, z)) \Rightarrow P(x, z)) \\ \forall x \forall y(P(x, y) \vee x = y \vee P(y, x)) \end{array} \right\}$$

Specify an interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{D} = \{d_1, \ldots, d_4\}$ and prove that $\mathcal{I} \models \Theta$ (i.e., $\mathcal{I} \models F$ for all $F \in \Theta$). Why is it not necessary to specify a variable assignment α to state a model of Θ?

(c) Are there also models of Θ with an infinite $\mathcal{D}$?

Exercise 11.2 (Substitutions and Unification)

(a) Compute the substitutions

(i) $P(x, y)\{\frac{x}{A}, \frac{y}{f(B)}\}$,
(ii) $P(x, y)\{\frac{x}{f(y)}\}\{\frac{y}{g(B,B)}\}$,
(iii) $P(x, y)\{\frac{x}{f(y)}, \frac{y}{g(B,B)}\}$, and
(iv) $P(x, y)\{\frac{z}{f(B)}, \frac{x}{A}\}$

(b) Apply the unification algorithm to the following set of literals: $\{R(h(x), f(h(u), y)), R(y, f(y, h(g(A))))\}$. In each step, give the values of T_k, s_k, D_k, v_k, and t_k.

Exercise 11.3 (Value iteration algorithm)

Consider the following grid world. The u values specify the utilities after convergence of the value iteration and r is the reward associated with a state. Assume that $\gamma = 1$ and that an agent can perform four possible actions: **North**, **South**, **East** und **West**. With probability 0.7 the agent reaches the intended state, with probability 0.2 it moves to the right of the intended direction, and with probability 0.1 to the left.

$u = 8$	$u = 15$	$u = 9$
$u = 2$	$r = 2$	$u = 7$
$u = 4$	$u = 16$	$u = 11$

Which is the best action an agent can execute if he is currently in the center state of the grid world? Justify your answer. Which utility does the center state have?

Exercise 11.4 (Policy iteration algorithm)

Let $\gamma = 0.5$ and let there be only the actions **East** and **West**. With probability 0.9 the agent reaches the intended state (or stays where he was, if the action would move him out of the grid), and with probability 0.1 he moves in the opposite direction. The reward in the three western states is -0.05 each.

s_0	s_1	s_2	s_3
$\longleftarrow$	$\longleftarrow$	$\longleftarrow$	$r = +1$

Perform one step of the policy iteration algorithm. The initial policy is given by the arrows in the states. Give the linear system of equations for the first policy evaluation, a solution to the system as well as the first improved policy π_1.

The exercise sheets may and should be handed in and be worked on in groups of three (3) students. Please fill the cover sheet[1] and attach it to your solution.

[1] `http://www.informatik.uni-freiburg.de/~ki/teaching/ss10/gki/coverSheet-english.pdf`