**Foundations of Artificial Intelligence**

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# Exercise Sheet 5

**Due: Tuesday, Juni 8, 2010**

**Exercise 5.1** (Conditional independence)

This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

(a) Suppose we wish to calculate $\mathbf{P}(X|E_1, E_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

(i) $\mathbf{P}(E_1, E_2)$, $\mathbf{P}(X)$, $\mathbf{P}(E_1|X)$, $\mathbf{P}(E_2|X)$

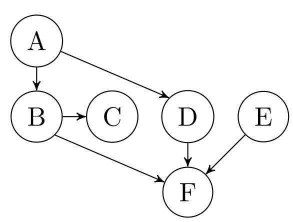
(ii) $\mathbf{P}(E_1, E_2)$, $\mathbf{P}(X)$, $\mathbf{P}(E_1, E_2|X)$

(iii) $\mathbf{P}(X)$, $\mathbf{P}(E_1|X)$, $\mathbf{P}(E_2|X)$

(b) Suppose we know that $\mathbf{P}(E_1|X, E_2) = \mathbf{P}(E_1|X)$ for all values of $X$, $E_1$, and $E_2$. Now which of the three sets are sufficient?

**Exercise 5.2** (Bayesian Networks)

Consider the following Bayesian network:




(a) Rewrite the joint probability distribution $P(A, B, C, D, E, F)$ using the conditional independencies expressed by the network.

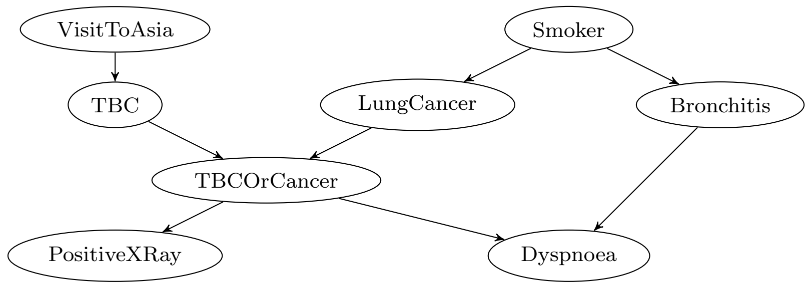
(b) Suppose that all the random variables $A, B, C, D, E, F$ in the Bayesian network can only have two possible values *yes* and *no*. What's the minimum number of probabilities required to fully define the Bayesian network whose structure is given above?

*Hint:* Remember that e.g. $P(E = yes) = 1 - P(E = no)$.

(c) How many probabilities would be required to define the full joint distribution over $A, B, C, D$, $E, F$ if we could not assume the conditional independencies expressed by the Bayesian network?

**Exercise 5.3** (Bayesian Networks)

Consider the following Bayesian network:




(a) Determine which of the following conditional independence statements follow from the structure of the Bayesian network ($Ind(U, V \mid W)$ denotes that $U$ is conditionally independent of $V$ given $W$, and $Ind(U, V)$ denotes unconditional independence of $U$ and $V$):

(i) $Ind(TBC, VisitToAsia)$

(ii) $Ind(VisitToAsia, Smoker)$

(iii) $Ind(VisitToAsia, PositiveXRay \mid TBCOrCancer)$

(iv) $Ind(VisitToAsia, Dyspnoea \mid TBCOrCancer)$

(v) $Ind(TBC, Smoker \mid PositiveXRay)$

(b) Compute $P(Dyspnoea|Smoker, \neg TBC)$. The relevant entries in the conditional probability tables are given below:

$$\begin{aligned}
P(LungCancer|Smoker) &= 0.1 \\
P(LungCancer|\neg Smoker) &= 0.01 \\
P(Bronchitis|Smoker) &= 0.2 \\
P(Bronchitis|\neg Smoker) &= 0.1 \\
P(TBCOrCancer|TBC, LungCancer) &= 1 \\
P(TBCOrCancer|TBC, \neg LungCancer) &= 1 \\
P(TBCOrCancer|\neg TBC, LungCancer) &= 1 \\
P(TBCOrCancer|\neg TBC, \neg LungCancer) &= 0 \\
P(Dyspnoea|TBCOrCancer, Bronchitis) &= 0.9 \\
P(Dyspnoea|TBCOrCancer, \neg Bronchitis) &= 0.7 \\
P(Dyspnoea|\neg TBCOrCancer, Bronchitis) &= 0.6 \\
P(Dyspnoea|\neg TBCOrCancer, \neg Bronchitis) &= 0.05
\end{aligned}$$

The exercise sheets may and should be handed in and be worked on in groups of three (3) students. Please fill the cover sheet[1] and attach it to your solution.

[1] http://www.informatik.uni-freiburg.de/~ki/teaching/ss10/gki/coverSheet-english.pdf