Foundations of Artificial Intelligence

Prof. Dr. B. Nebel, Prof. Dr. W. Burgard
B. Frank, A. Karwath, G. Röger
Summer Term 2009

University of Freiburg
Department of Computer Science

Exercise Sheet 8

Due: Tuesday, June 30, 2009

Exercise 8.1 (Bayesian Rule)

Assume, you are at night in Athens and witness a car accident in which a taxi is involved. 90% of the taxis in Athens are green, all others are blue. You are absolutely sure that the taxi involved in the accident was blue. But tests show that distinguishing between blue and green at darkness is only 75% reliable. If you take this into consideration, what is the probability that the taxi was really blue? (Hint: Distinguish exactly between the statement that a taxi *is* red and the statement that a taxi *appears* to be red.)

Exercise 8.2 (Conditional probabilities)

Suppose you are given a bag containing n unbiased coins, out of which $n - 1$ are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

(a) Suppose you reach into the bag, pick out a coin uniformly at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?

(b) Suppose you continue flipping the coin for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin?

(c) Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns FAKE if all k flips come up heads, otherwise it returns NORMAL. What is the (unconditional) probability that this procedure makes an error?

Exercise 8.3 (Conditional independence)

This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

(a) Suppose we wish to calculate $\mathbf{P}(X|E_1, E_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

  (i) $\mathbf{P}(E_1, E_2)$, $\mathbf{P}(X)$, $\mathbf{P}(E_1|X)$, $\mathbf{P}(E_2|X)$

  (ii) $\mathbf{P}(E_1, E_2)$, $\mathbf{P}(X)$, $\mathbf{P}(E_1, E_2|X)$

  (iii) $\mathbf{P}(X)$, $\mathbf{P}(E_1|X)$, $\mathbf{P}(E_2|X)$

(b) Suppose we know that $\mathbf{P}(E_1|X, E_2) = \mathbf{P}(E_1|X)$ for all values of X, E_1, and E_2. Now which of the three sets are sufficient?

The exercise sheets may and should be handed in and be worked on in groups of three (3) students. Please fill the cover sheet[1] and attach it to your solution.

[1] `http://www.informatik.uni-freiburg.de/~ki/teaching/ss09/gki/coverSheet-english.pdf`